Large N volume reduction of Minimal Walking Technicolor

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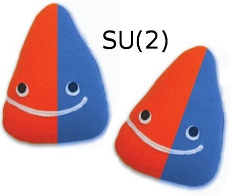
Cartoon Outline of Talk



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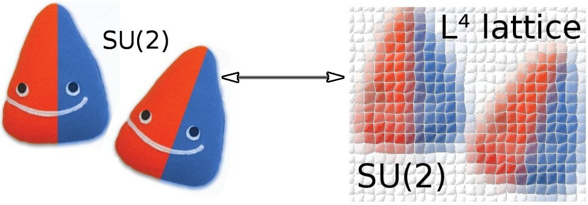

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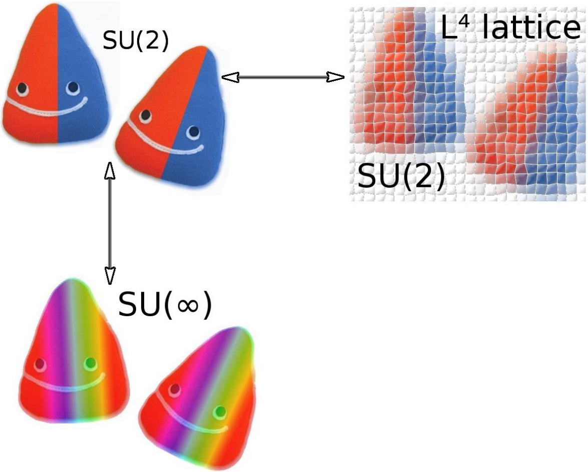

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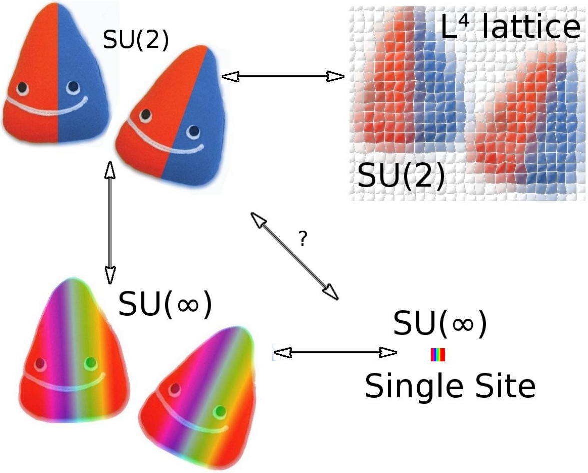
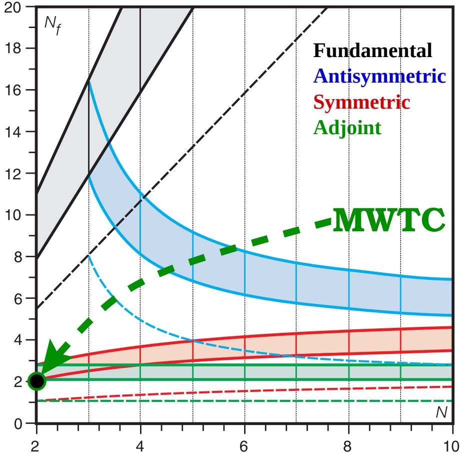


Dynamical Electroweak Symmetry Breaking



[arXiv:hep-ph/0611341]

- Dynamical EWSB or Technicolor Models
- In particular MWT: 2 dirac fermions transforming under the adjoint representation of SU(2)

Saninno, Tuominen
[arXiv:hep-ph/0405209]

Mass Anomalous Dimension

Size of quark mass terms in the effective action depend on the value of the anomalous mass dimension γ.

Quark Masses

$$\frac{\langle\overline{\Psi}\Psi\rangle_{ETC}}{\Lambda_{ETC}^2}\overline{\psi}\psi$$

Power Enhancement

$$\langle\overline{\Psi}\Psi\rangle_{ETC} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma}\langle\overline{\Psi}\Psi\rangle_{TC}$$

- Need $\gamma \simeq 1$ to generate large enough quark masses.
- Important quantity to measure in TC models.

Why Large N?

- In perturbation theory, first two universal coefficients predict γ_* is independent of N, so we expect the large N value to be close to the $N = 2$ value.
- At large N the theory is (under certain conditions) volume independent, so the calculation can be done on a small lattice or even a single site.
- Interesting cross check of method, perturbation theory and large N volume reduction.

Large–N Volume Independence

Eguchi-Kawai '82

In the limit $N_c \to \infty$, the properties of U(N_c) Yang–Mills theory on a periodic lattice are independent of the lattice size.

$$S_{YM} = S_{EK} \equiv N_c b \sum_{\mu<\nu} Tr \left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

where $b = \frac{1}{\lambda} = \frac{1}{g^2 N_c}$ is the inverse bare 't Hooft coupling, held fixed as $N_c \to \infty$.

Conditions

...but it turns out only

- for single–trace observables defined on the original lattice of side L, that are invariant under translations through multiples of the reduced lattice size L'
- and if the $U(1)^d$ center symmetry is not spontaneously broken, i.e. on the lattice the trace of the Polyakov loop vanishes.

Twisted Eguchi–Kawai

Gonzalez–Arroyo Okawa '83

Impose twisted boundary conditions, such that the classical minimum of the action preserves a Z_N^2 subgroup of the center symmetry.

$$S_{TEK} = N_c b \sum_{\mu<\nu} Tr\left(z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c.\right)$$

$$\text{where } z_{\mu\nu} = exp\{2\pi i k/\sqrt{N}\}$$

- Original choice is $k = 1$

Twisted Eguchi–Kawai

- Original choice $k = 1$ seen to break center–symmetry at intermediate couplings for $N \gtrsim 100$
- But symmetry can be restored by scaling the twist k with N

Gonzalez–Arroyo Okawa [arXiv:1005.1981]

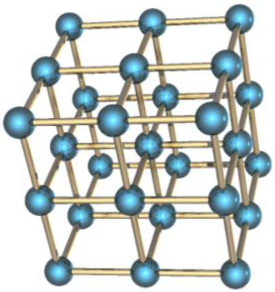
QCDadj

Kotvul Unsal Yaffe '07

Add (massless or light) adjoint fermions with periodic boundary conditions

- Preserves center symmetry down to a single site
- Works in perturbation theory (for $am \lesssim \frac{1}{N}$)
- And in lattice simulations (even for $am \lesssim 1$)

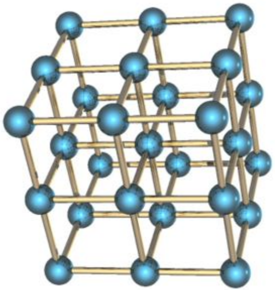
Lattice Field Theory



Formulate field theory on a discrete set of space–time points:

- Physical volume $L^4 = (\hat{L}a)^4$
- $\hat{L}^4$ points, lattice spacing a
- Quarks live on sites
- Gauge fields live on links between sites
- Simulate on a big computer

Continuum Limit



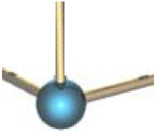
Lattice provides regularisation:

- UV cut–off: $1/a$
- IR cut–off: $1/L$

To recover continuum theory:

- Take $1/L \to 0$ limit ($\hat{L} \to \infty$)
- Take $a \to 0$ limit ($b \to \infty$)

Twisted Reduction



A single site lattice:

- Single lattice site instead of $\hat{L}^4$ points
- Equivalent to $L^4 = (\sqrt{N}a)^4$ lattice
- Can substitute $\sqrt{N}$ with $\hat{L}$
- Then everything else is the same

Mode Number Method

At small eigenvalues, at leading order,

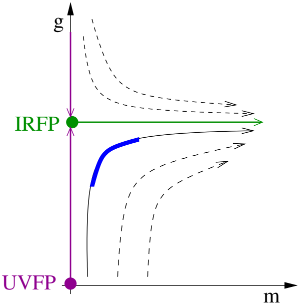
Spectral density of the Dirac Operator

$$\rho(\omega) \propto \mu^{\frac{4\gamma_*}{1+\gamma_*}} \omega^{\frac{3-\gamma_*}{1+\gamma_*}} + \dots$$

- Integral of this is the mode number, which is just counting the number of eigenvalues of the Dirac Operator on the lattice.
- Fitting this to the above form can give a precise value for γ, as done recently for MWT by Agostino Patella.

Patella [arXiv:1204.4432]

Mode Number Fit Range



RG flows in mass–deformed CFT:

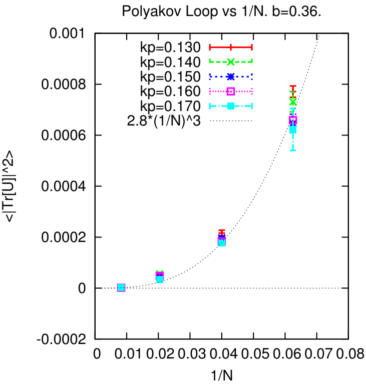
- Flow from UV (high eigenvalues) to IR (low eigenvalues)
- Finite mass drives us away from FP in the IR
- Interested in intermediate blue region
- $\frac{1}{\sqrt{N}} \ll m \ll \overline{\Omega}_{IR} < \Omega < \overline{\Omega}_{UV} \ll \frac{1}{a}$

Simulation Details

- Simulate large N version of MWT.
  - SU(N) gauge theory with 2 light adjoint Dirac fermions with periodic boundary conditions.
- Use single site 1^4 lattices with N up to 289.
  - $V_{eff} = N^2$, so equivalent to $L^4 = 17^4$.
- Measure lowest 1000 eigenvalues of the Dirac operator Q^2.
- Choose bare lattice coupling $b = 1/\lambda = 0.35, 0.36$.
  - Need to stay in weak coupling phase.
  - But want fairly strong coupling to minimise $1/N$ effects.

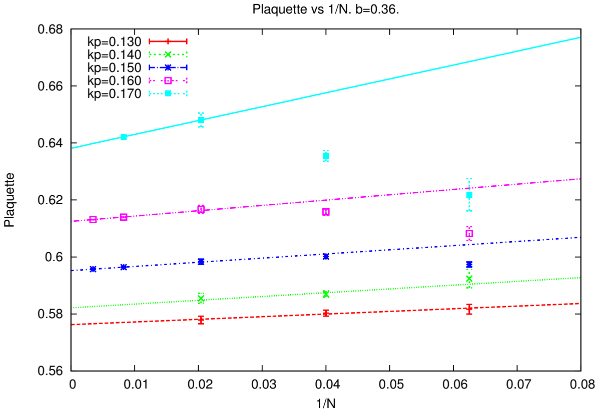
Polyakov Loop

Polyakov loop is zero up to 1/N corrections, so reduction holds.



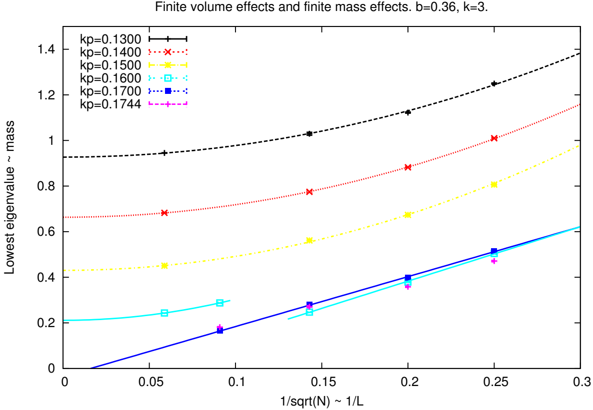
Plaquette vs 1/N

Plaquette: see larger finite–N effects for lighter masses.



Lowest Dirac Eigenvalue vs 1/N

Lowest eigenvalue has two distinct regimes.



Large volume vs small volume

- Large volume regime (p–regime)
  - $mL \gg 1$
  - $\lambda = m + c/N$
  - Can perform mode number fit
- Small volume regime (ϵ–regime)
  - $mL \ll 1$
  - $\lambda \sim 1/L$
  - Comparison to chiral random matrix theory?
  - Also mode number fit if affected eigenvalues are excluded from the fit?

Method

Fit data to the function

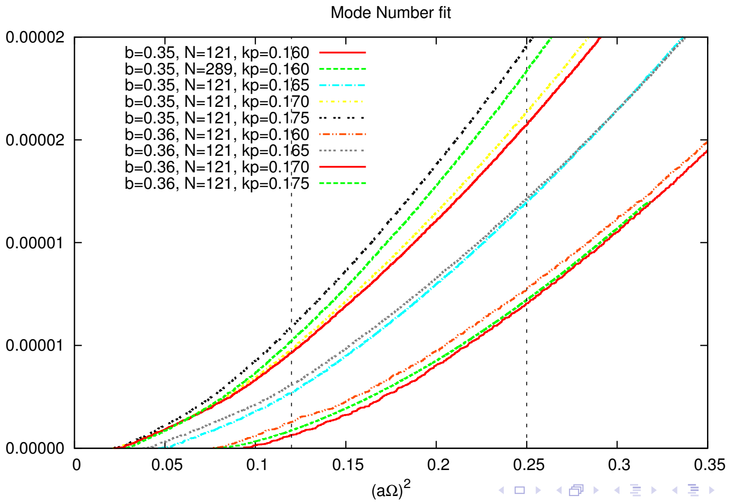
$$a^{-4}\overline{\nu}(\Omega) \simeq a^{-4}\overline{\nu}_0 + A\left[(a\Omega)^2 - (am)^2\right]^{\frac{2}{1+\gamma_*}}$$

in some intermediate range $a\Omega_L < a\Omega < a\Omega_H$ where

- $a^{-4}\overline{\nu}(\Omega)$ is the number of eigenvalues of Q^2 below Ω^2 divided by the volume
- $a^{-4}\overline{\nu}_0$ is a fitted parameter (contribution of small excluded eigenvalues, $\propto M_{PS}^4$)
- am is a fitted parameter (physical mass)
- A is a fitted parameter

Patella [arXiv:1204.4432]

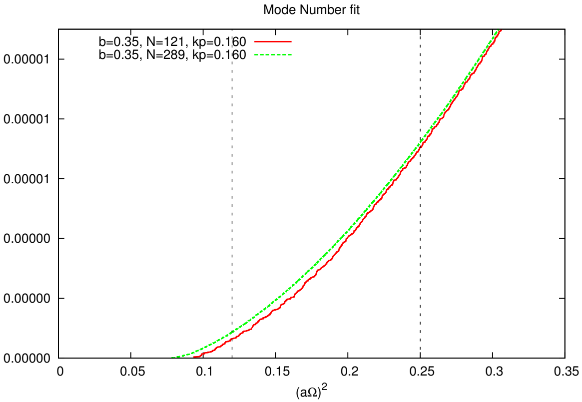
Mode Number Data



Mode Number Example Fit $b = 0.35, \kappa = 0.16$

$N = 289$: $A = 1.11 \times 10^{-4}, am = 0.271, \gamma = 0.267$

$N = 121$: $A = 1.25 \times 10^{-4}, am = 0.296, \gamma = 0.255$



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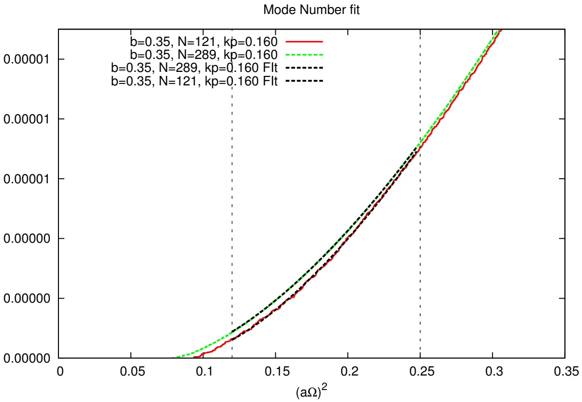
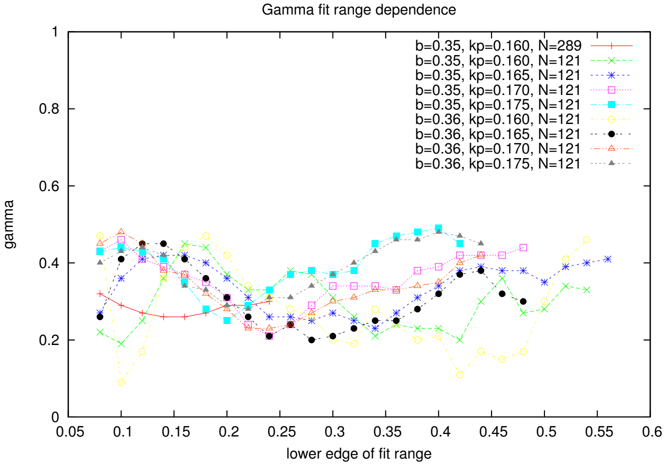


Mode Number Fit Range [preliminary]



Conclusion and Future Work

- Promising initial results.
  - Volume reduction seems to work
  - Finite volume and finite mass effects understood
  - Preliminary results give $\gamma \simeq 0.2 - 0.4$
- Would be very interesting to compare with $n_f = 1$
- Also need to investigate fully the systematics of the fitting procedure.
- And want to try different twist and couplings, larger N, lighter masses.

Extra Slides

blah blah