

MCRG Minimal Walking Technicolor

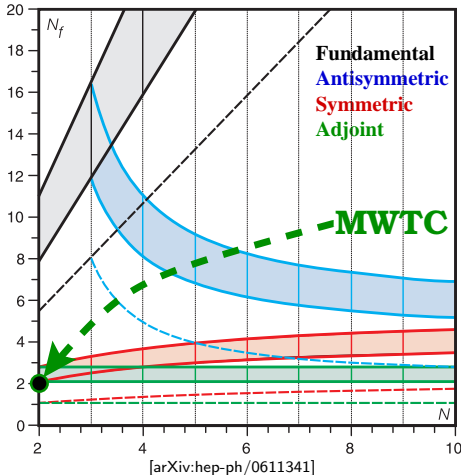
Liam Keegan

June 2010

Edinburgh University

Simon Catterall, Luigi Del Debbio, Joel Geidt

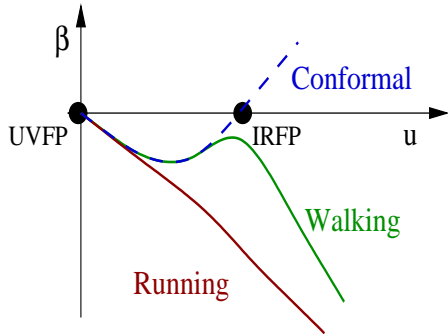
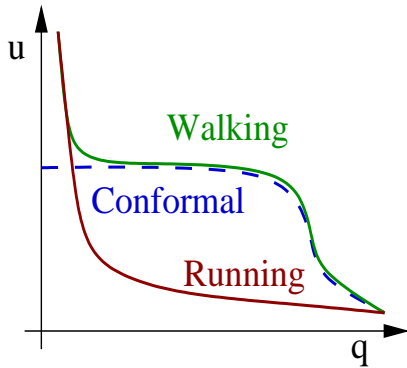
Minimal Walking Technicolor



- Simplest interesting model: MWT
- 2 dirac fermions transforming under the adjoint representation of $SU(2)$

Saninno, Tuominen
[arXiv:hep-ph/0405209]

Walking Technicolor Cartoon



Scheme dependence

- Walking/Running of coupling is scheme dependent
- Want to measure physical, scheme independent quantities:
 - **Existence** of fixed point
 - **Anomalous mass dimension** at the fixed point

Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

$$\hat{\xi}^{(0)} \quad , \quad \{g_i^{(0)}\}$$

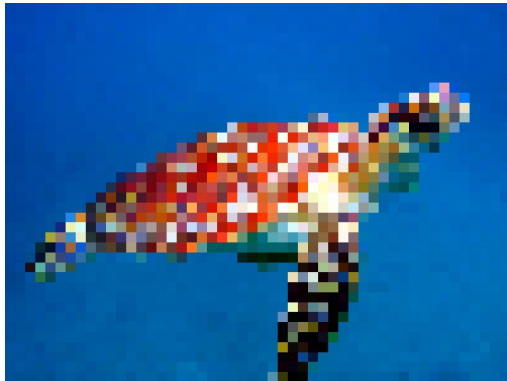
Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
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- Look at evolution of all couplings

$$\hat{\xi}^{(1)} = \hat{\xi}^{(0)} / 2 \quad , \quad \{g_i^{(1)}\}$$

Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

$$\hat{\xi}^{(2)} = \hat{\xi}^{(0)} / 2^2 \quad , \quad \{g_i^{(2)}\}$$

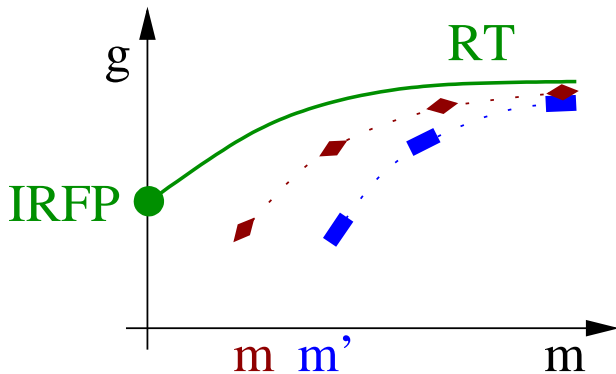
Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
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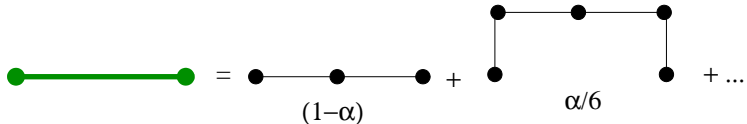
$$\hat{\xi}^{(3)} = \hat{\xi}^{(0)} / 2^3 \quad , \quad \{g_i^{(3)}\}$$

Monte Carlo Renormalisation Group



- Match after $n(n-1)$ steps
- $s = 2$ change in scale
- Step scaling of bare couplings

Lattice Blocking Transform



- Free parameter α adjusts RG blocking transform
- Optimise α to approach RT quickly such that subsequent steps give the same matching

$$V_{n,\mu} = \text{Proj} \left[(1 - \alpha) U_{n,\mu} U_{n+\mu,\mu} + \frac{\alpha}{6} \sum_{\nu \neq \mu} U_{n,\nu} U_{n+\nu,\mu} U_{n+\mu+\nu,\mu} U_{n+2\mu,\nu}^\dagger \right]$$

MCRG Key Points

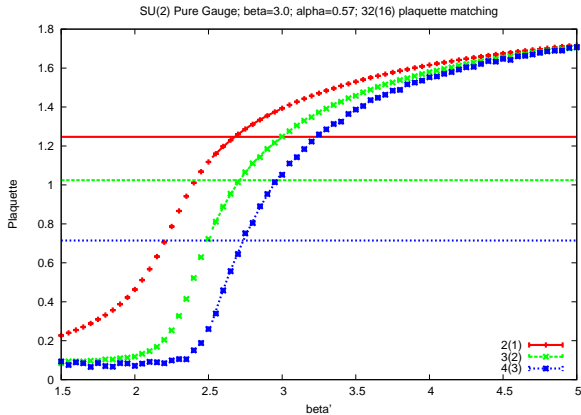
- Find pairs of couplings with identical actions, but whose correlation lengths differ by a factor 2
- Identify matching actions by comparing observables on blocked lattices (plaquette and 6-link loops)
- Always match between lattices with the same number of points to minimise finite size errors
- Optimise α to approach RT quickly so that subsequent steps give the same matching

Hasenfratz [arXiv:hep-lat/0907.0919]

Pure Gauge Simulation

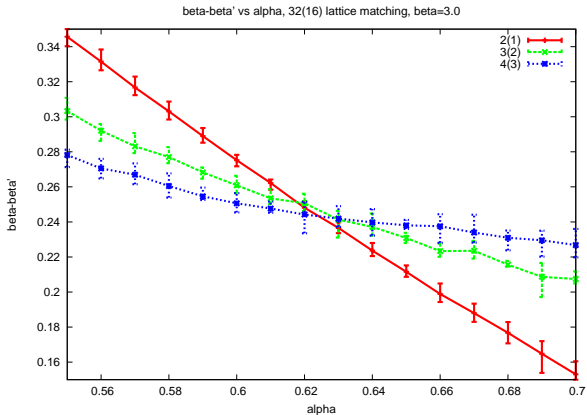
- Simulated on lattices of size $L=32,16$
- Allows for 3 matchings; 2(1), 3(2), 4(3) steps on the $32^4(16^4)$ lattices
- Optimise α such that these steps predict the same matching coupling

Plaquette Matching



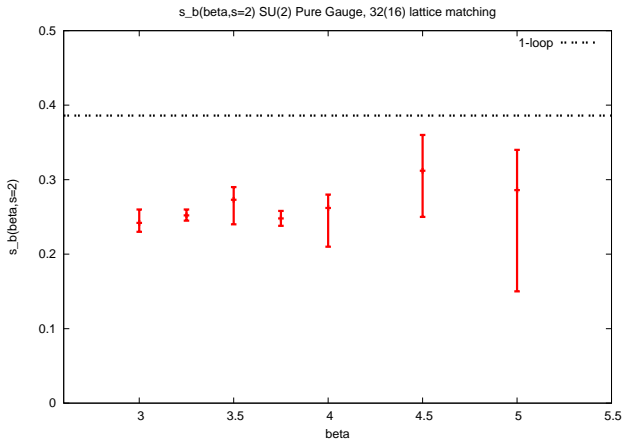
- 32(16) matching
- $\beta = 3.0$
- $\alpha = 0.57$

Alpha Optimisation

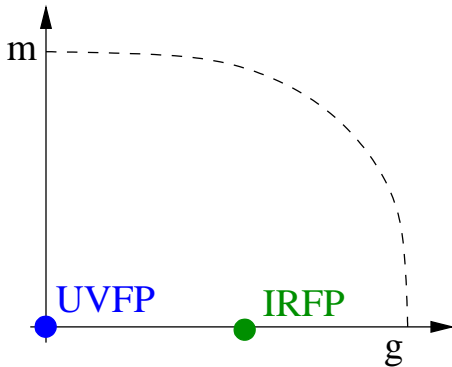


- $\beta = 3.0$
- $\alpha_{opt} \sim 0.63$
- $s_b = 0.24(2)$

Pure Gauge Bare Step Scaling



Phase diagram

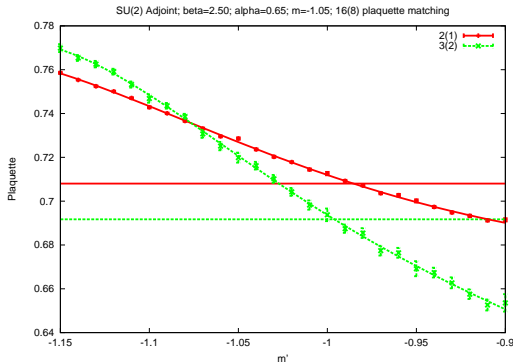


- UVFP: both m and g are relevant
- IRFP: m relevant, g irrelevant
- Near IRFP can match in m , value of g should be irrelevant

Simulation details

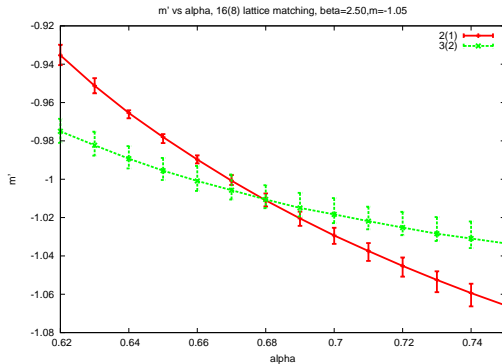
- Simulated on lattices of size $L=16,8$
- Allows for 2 matchings; 2(1), 3(2) steps on the $16^4(8^4)$ lattices
- Keep β constant, match in bare mass
- Optimise α such that these all agree to find continuum physics

Plaquette Matching



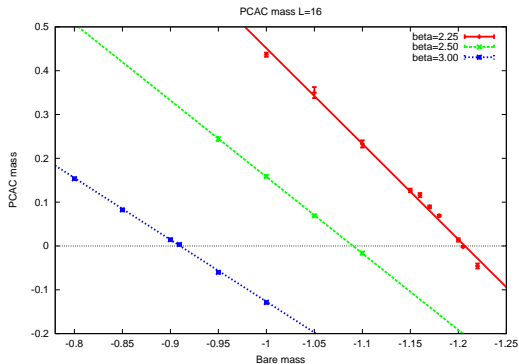
- 16^4 blocked **two/three** times
- Single mass $m = -1.05$
- 8^4 blocked **one/two** times
- Many masses $-1.15 < m' < -0.90$

Alpha Optimisation



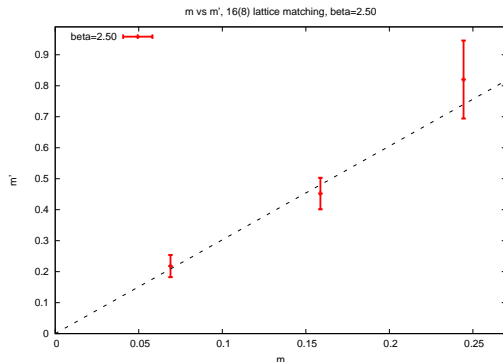
- $\alpha_{opt} \sim 0.68$
- $m = -1.05$
- $m' = -1.01(2)$

PCAC Masses



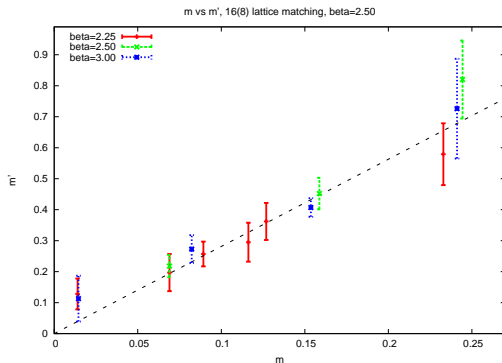
- Have matching bare masses, but additively renormalised quantities
- So need to convert to PCAC masses to be able to extract anomalous dimension

Anomalous Dimension



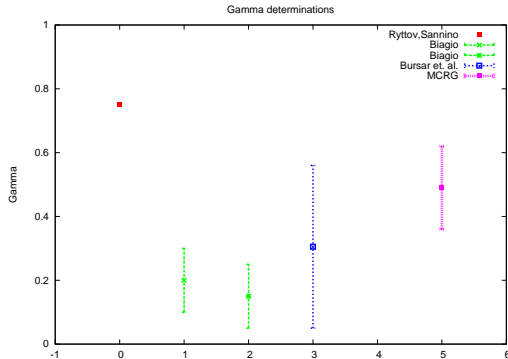
- Extract γ from ratio of masses:
- $m' = 2^{\gamma+1}m$
- To verify that beta is irrelevant, repeat at different beta...
- Linear fit gives $\gamma = 0.49(13)$

Anomalous Dimension



- Extract γ from ratio of masses:
- $m' = 2^{\gamma+1} m$
- To verify that beta is irrelevant, repeat at different beta...
- Linear fit gives $\gamma = 0.49(13)$

Summary



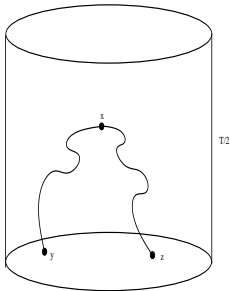
- We find $\gamma = 0.49(13)$
- All-order prediction $\gamma = 0.75$ has $\chi^2/dof \sim 2.3$

Rytov, Sannino [arXiv:hep-th/0711.3745], Biagio [arXiv:hep-ph/0911.0020], Bursar et. al. [arXiv:hep-ph/0910.4535]

Future Plans

- Try different RG blocking transforms, look for universality.
- Use 32^4 lattices, would give 3 matching steps instead of 2.
- Match in more observables, including fermionic ones.
- Look for fixed point in coupling by matching in β at zero mass

PCAC Mass



PCAC mass is defined using the Partially Conserved Axial Current:

PCAC Mass

$$am(x_0) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*)f_A(x_0)}{2f_P(x_0)}$$

$$f_A(x_0) = -1/12 \int d^3y d^3z \langle \bar{\psi}(x_0) \gamma_0 \gamma_5 \tau^a \psi(x_0) \bar{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle$$

$$f_P(x_0) = -1/12 \int d^3y d^3z \langle \bar{\psi}(x_0) \gamma_5 \tau^a \psi(x_0) \bar{\zeta}(y) \gamma_5 \tau^a \zeta(z) \rangle$$

Prediction for anomalous dimension

Conjectured all orders beta function

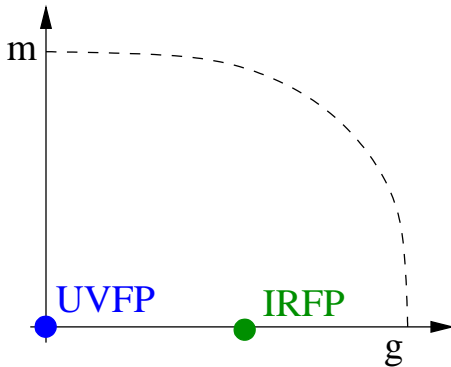
$$\beta(g) = \frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3} T(r) N_f \gamma(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta'_0}{\beta_0}\right)}$$

$$\beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} T(r) N_f, \quad \beta'_0 = C_2(G) - T(r) N_f$$

- For MWTC this predicts anomalous dimension $\gamma = 3/4$ at fixed point
- This is a scheme-independent quantity at a fixed point

Ryttov, Sannino [arXiv:0711.3745]

Phase diagram

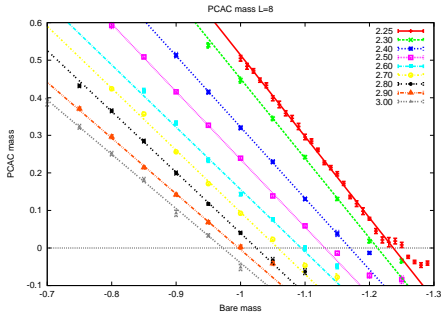


- UVFP: both m and g are relevant
- IRFP: m relevant, g irrelevant
- Can try tuning the mass to zero
- Then measure the scaling of the least irrelevant operator, hopefully g

Simulation details

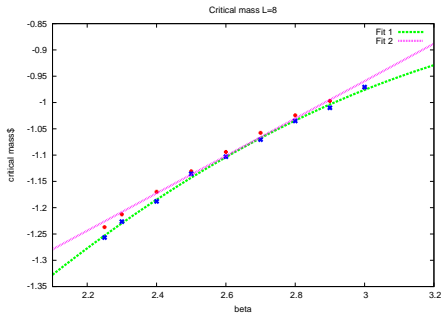
- Simulated on lattices of size $L = 16, 8$
- Allows for 2 matchings; 2(1), 3(2) steps on the $16^4(8^4)$ lattices
- Tune all runs to the critical $m_{PCAC} = 0$ massless point
- Optimise α such that these all agree to find continuum physics

Mass tuning



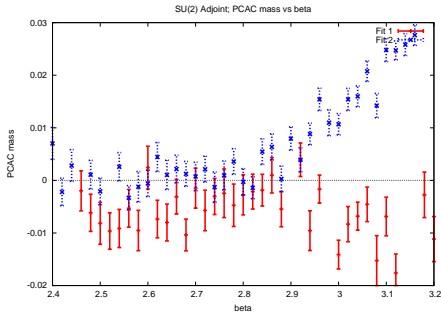
- Measure m_{PCAC} for a range of β, m
- Interpolate to find the m_{crit} for each β
- Simulate at m_{crit} for each β
- Two different sets of runs to see mass dependence

Mass tuning



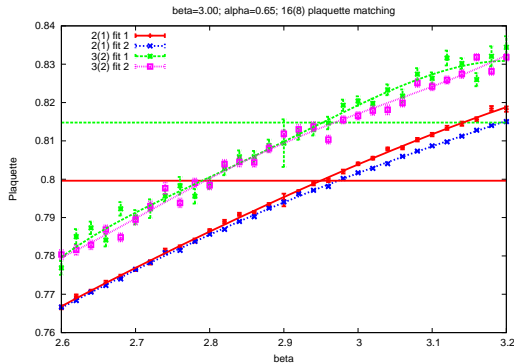
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Mass tuning



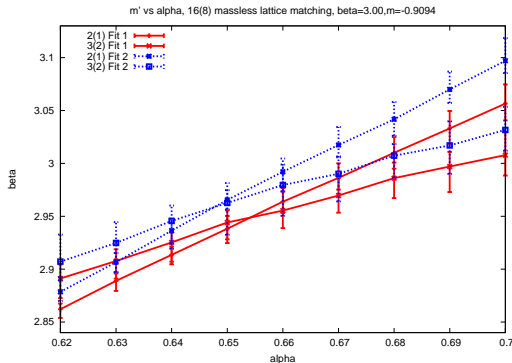
- Measure m_{PCAC} for a range of β, m
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Plaquette Matching



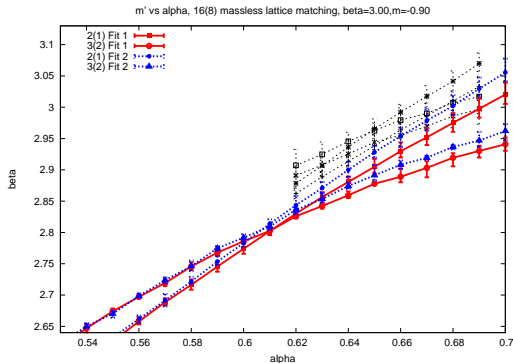
- Massless $L = 16$
- $m_{PCAC} = 0.003(1)$
- Different fits show sensitivity to $L=8$ mass tuning

Alpha Optimisation



- Looks less sensitive to $L=8$ mass tuning than expected.
- $s_b = 0.04(7)$
- $s_b = 0.05(7)$
- Step scaling consistent with zero within errors.

Mass sensitivity



- Near-massless $L = 16$
- $m_{PCAC} = 0.015(1)$
- $s_b = 0.20(6)$
- $s_b = 0.20(7)$
- Very sensitive to $L=16$ mass tuning

Particle content of MWT

- Fermionic content:
 - (U,D) techni-quark doublet
 - (N,E) new lepton doublet
 - Composite techniquark-technigluon doublet
- Composite Higgs from techni-pion

MWT LHC Phenomenology

- Details depend on choice of ETC model
- Then construct low energy EFT for LHC

Frandsen, Sannino, et. al. [arXiv:0710.4333v1] [arXiv:0809.0793v1]

MWT Dark Matter candidate

- Lightest technibaryon is a cold dark matter candidate
- TIMP: Technicolour Interacting Massive Particle
- iTIMP: lightest weak isotriplet technibaryon
- Prospects for discovery/exclusion from both dark matter experiments and LHC

Frandsen, Sannino [arXiv:0911.1570]