Systematic Errors of the MCRG Method

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July 2011

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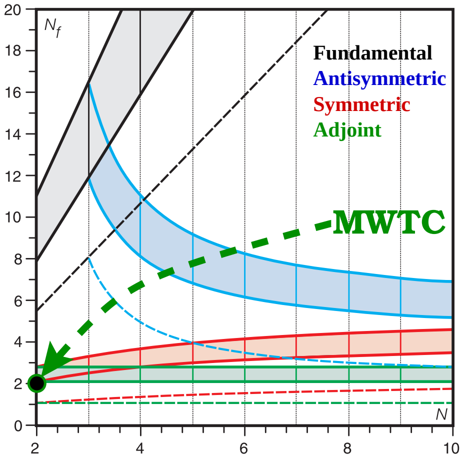
Minimal Walking Technicolor

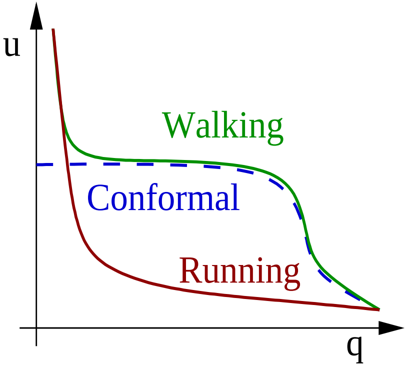


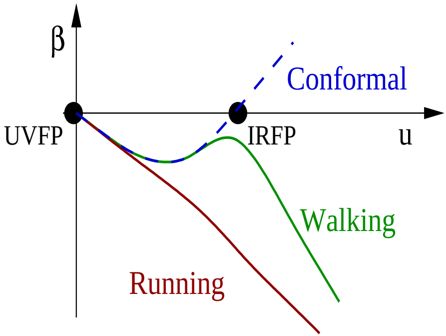
[arXiv:hep-ph/0611341]

- Simplest interesting model: MWT
- 2 dirac fermions transforming under the adjoint representation of SU(2)

Saninno, Tuominen
[arXiv:hep-ph/0405209]

Walking Technicolor Cartoon

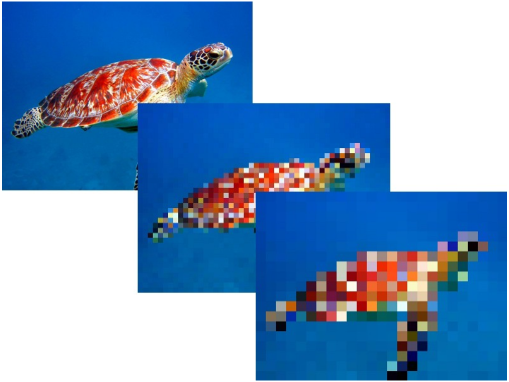




Scheme dependence

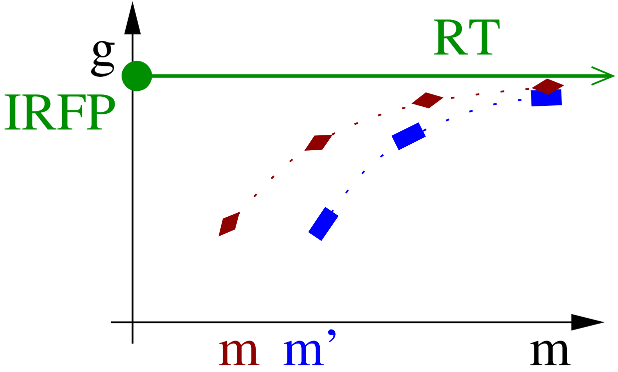
- Walking/Running of coupling is scheme dependent
- Want to measure physical, scheme independent quantities:
  - **Existence** of fixed point
  - **Anomalous mass dimension** at the fixed point

Wilson Renormalisation Group



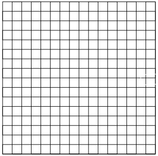
- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

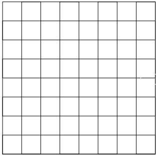
Monte Carlo Renormalisation Group



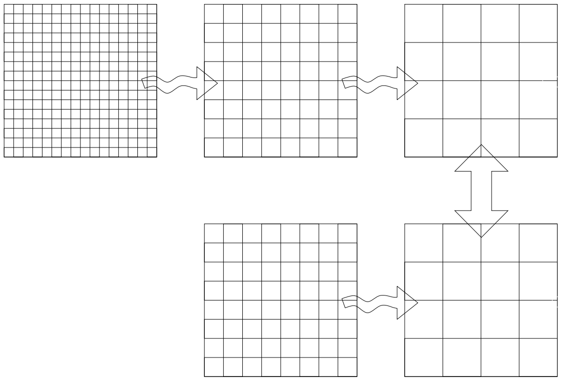
- Match after $n(n-1)$ steps
- $s = 2$ change in scale
- Step scaling of bare couplings

2–Lattice Matching





2–Lattice Matching



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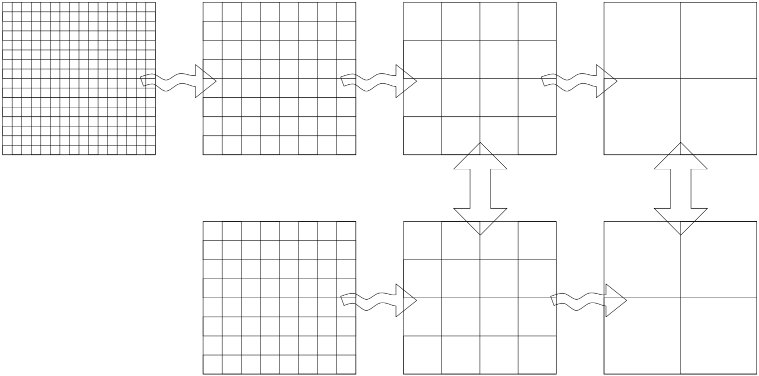
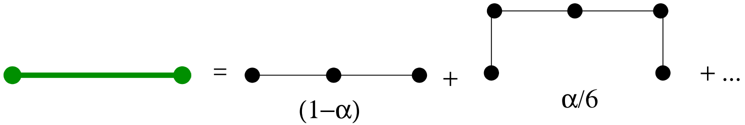


Lattice Blocking Transform



- Free parameter α adjusts RG blocking transform
- Optimise α to approach RT quickly such that subsequent steps give the same matching

$$V_{n,\mu} = Proj\left[(1-\alpha)U_{n,\mu}U_{n+\mu,\mu} + \frac{\alpha}{6}\sum_{\nu\neq\mu} U_{n,\nu}U_{n+\nu,\mu}U_{n+\mu+\nu,\mu}U^{\dagger}_{n+2\mu,\nu}\right]$$

MCRG Key Points

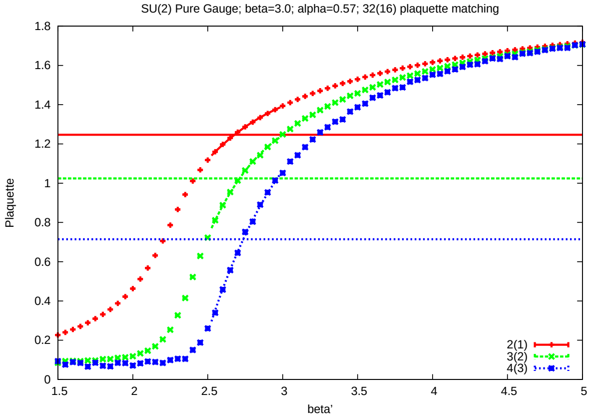
- Find pairs of couplings with identical blocked actions, whose correlation lengths differ by a factor 2
- Identify matching actions by comparing observables on blocked lattices (plaquette, 6–link and 8–link loops)
- Always match between lattices with the same number of points to minimise finite size errors
- Optimise α to approach RT quickly so that subsequent steps give the same matching

Hasenfratz [arXiv:hep-lat/0907.0919]

Pure Gauge Simulation

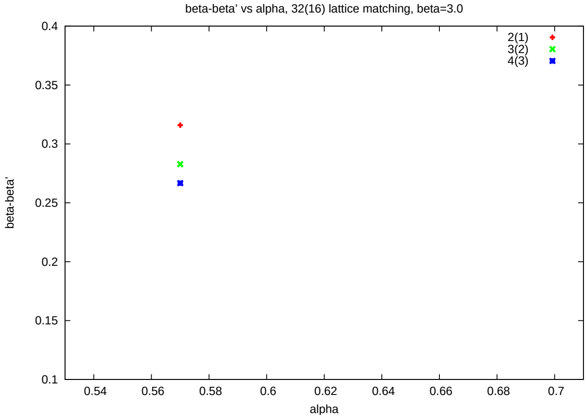
- Simulated on lattices of size L=32,16
- Allows for 3 matchings; 2(1), 3(2), 4(3) steps on the $32^4(16^4)$ lattices
- Optimise α such that these steps predict the same matching coupling

Plaquette Matching



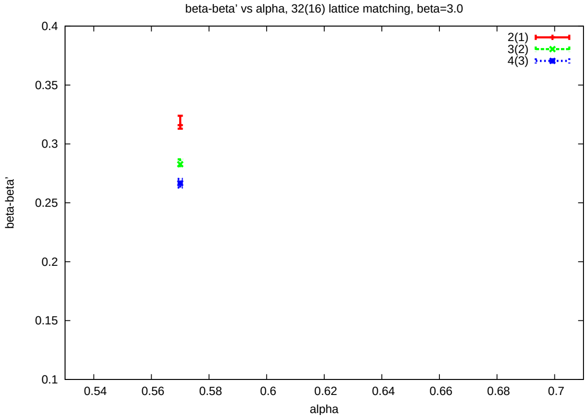
- 32(16) matching
- $\beta = 3.0$
- $\alpha = 0.57$

Alpha Optimisation



- $\beta = 3.0$
- $\alpha_{opt} \sim 0.63$
- $s_b = 0.24(2)$

Alpha Optimisation



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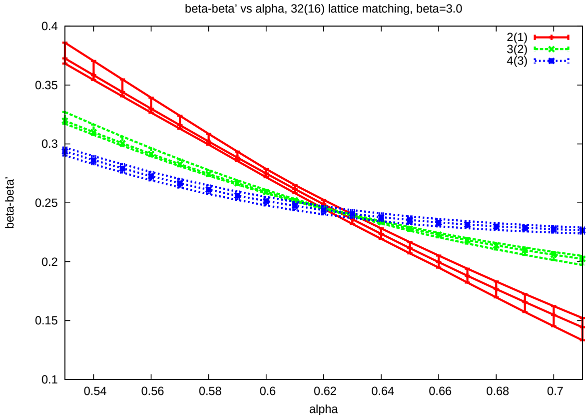

- $\beta = 3.0$
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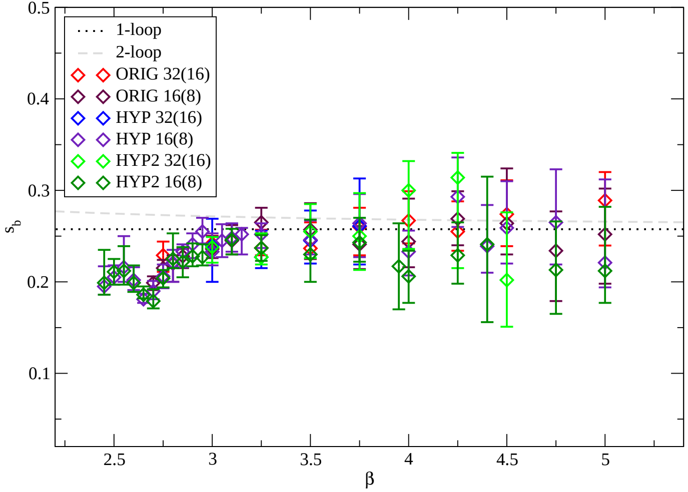
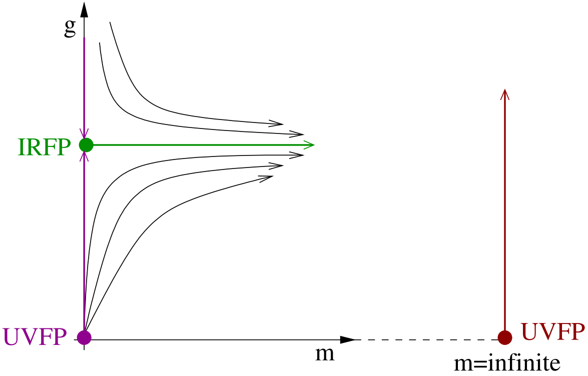
- $s_b = 0.24(2)$

Pure Gauge Bare Step Scaling



Phase diagram of full theory

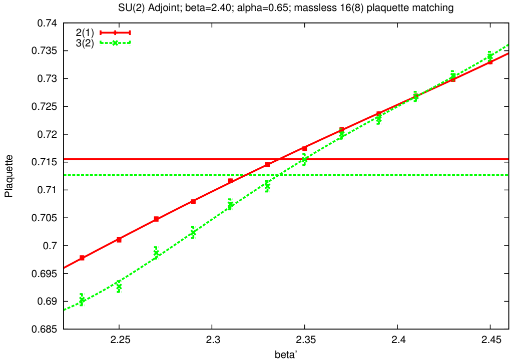


- At IRFP:
- Tune relevant m to zero
- Match in 'least-irrelevant' coupling g

Simulation details

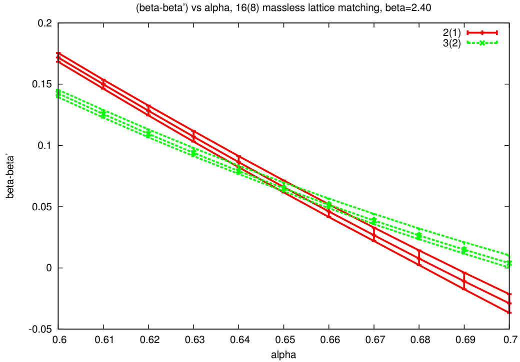
- Simulated on lattices of size L=16,8
- Allows for 2 matchings; 2(1), 3(2) steps on the $16^4(8^4)$ lattices
- Tune all PCAC masses to zero, then match in β
- Optimise α such that these all agree to find continuum physics

Plaquette Matching



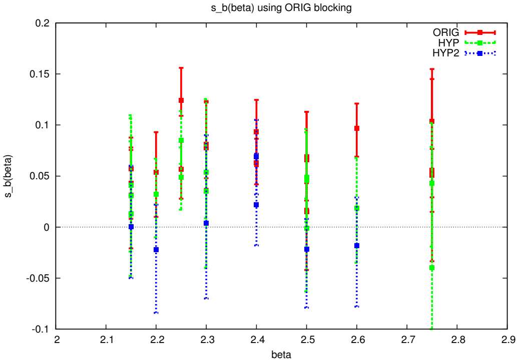
- 16^4 blocked two/three times
- Single $\beta = 2.40$
- 8^4 blocked one/two times
- Many β' values

Alpha Optimisation



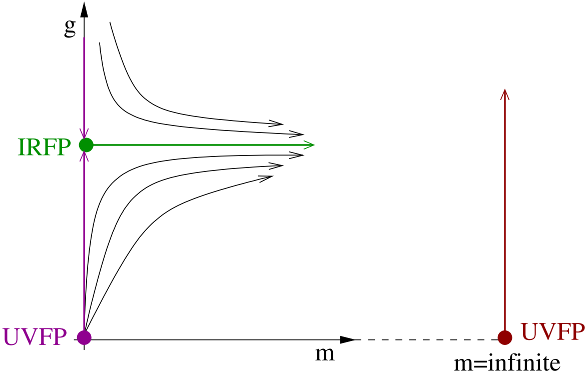
- $\alpha_{opt} \sim 0.65$
- $\beta = 2.40$
- $\beta' = 2.34(3)$
- $s_b = 0.06(3)$

Coupling Step Scaling



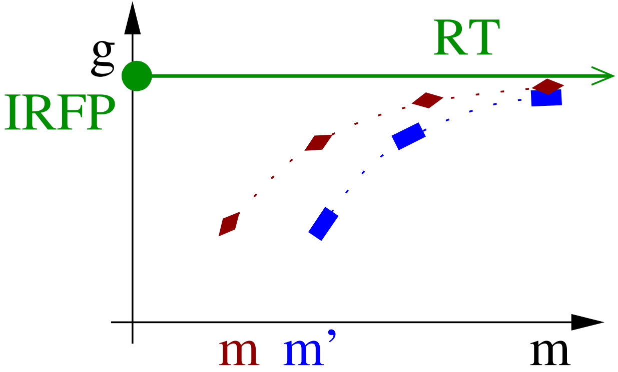
- FP would be indicated by change of sign in s_b
- Data compatible with change of sign
- But errors too large to identify a FP

Phase diagram



- At IRFP:
- m relevant
- g irrelevant
- Near IRFP can match in m, value of g should be irrelevant

Phase diagram

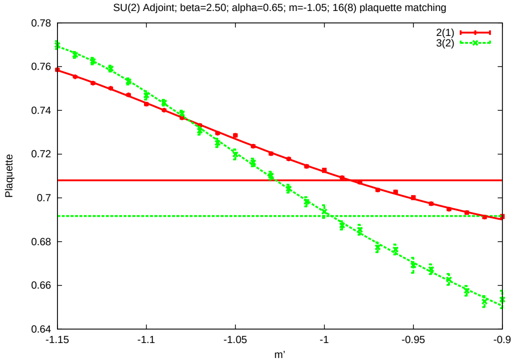


- At IRFP:
- *m* relevant
- *g* irrelevant
- Near IRFP can match in m, value of g should be irrelevant

Simulation details

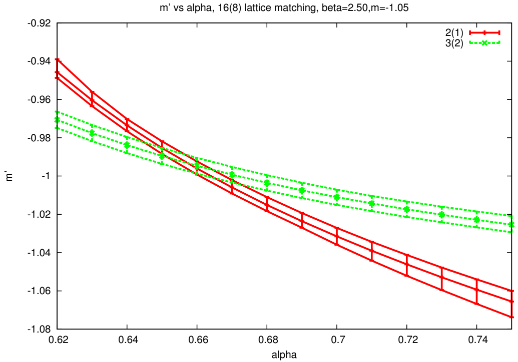
- Simulated on lattices of size L=16,8
- Allows for 2 matchings; 2(1), 3(2) steps on the $16^4(8^4)$ lattices
- Keep β constant, match in bare mass
- Optimise α such that these all agree to find continuum physics

Plaquette Matching



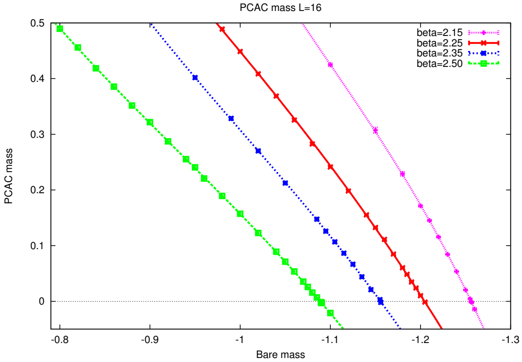
- 16^4 blocked two/three times
- Single mass $m = -1.05$
- 8^4 blocked one/two times
- Many masses $-1.15 < m' < -0.90$

Alpha Optimisation



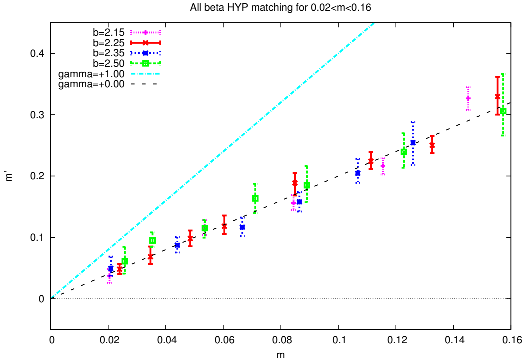
- $\alpha_{opt} \sim 0.68$
- $m = -1.05$
- $m' = -1.01(2)$

PCAC Masses



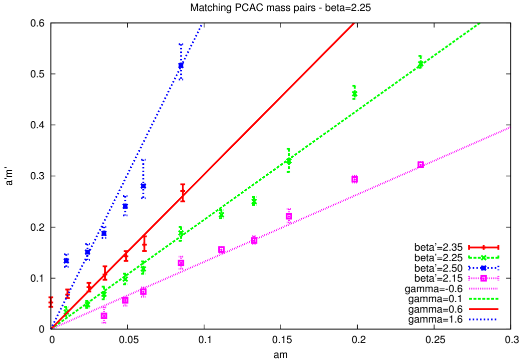
- Have matching bare masses, but additively renormalised quantities
- So need to convert to PCAC masses to be able to extract anomalous dimension

Anomalous Dimension



- Extract γ from ratio of masses:
- $m' = 2^{\gamma+1} m$
- Linear fit gives $\gamma = -0.03(13)$

Systematic Uncertainty



- $\beta = 2.25$
- β' uncertainty is a large systematic error
- $\beta' = 2.35 \rightarrow \gamma \sim 0.6$
- $\beta' = 2.25 \rightarrow \gamma \sim 0.0$
- $\beta' = 2.15 \rightarrow \gamma \sim -0.6$

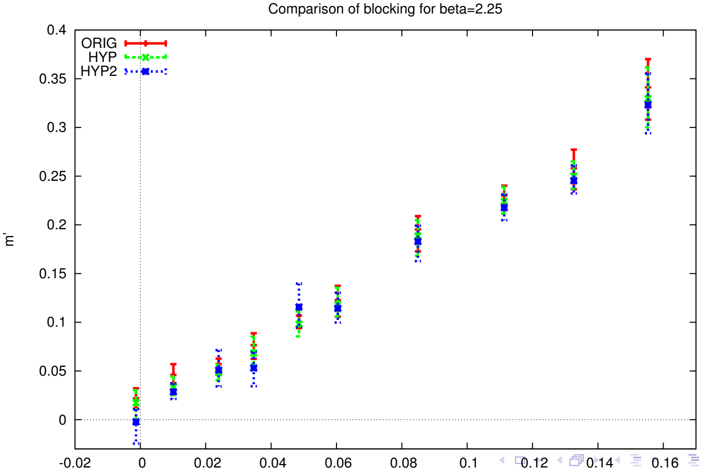
MCRG Conclusion

- Running of the coupling is slow, and consistent with walking and a fixed point
  - But our data cannot distinguish the two cases
- Mass anomalous dimension is small, but with large systematics
  - Mainly due to not finding a unique matching coupling

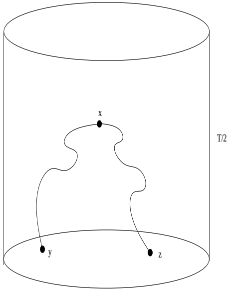
Future Plans

- Use 3 matching steps instead of 2
- Match in more observables, including fermionic ones
- Use stability matrix method to determine critical exponents
- All of these require larger lattices (32^4)

Blocking comparison



PCAC Mass



PCAC mass is defined using the Partially Conserved Axial Current:

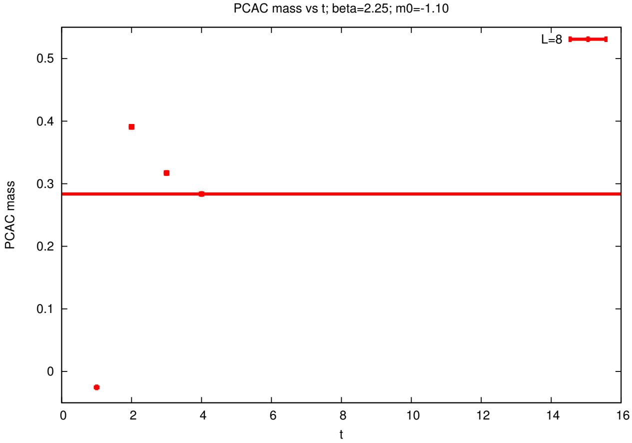
PCAC Mass

$$am(x_0) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*) f_A(x_0)}{2 f_P(x_0)}$$

$$f_A(x_0) = -1/12 \int d^3y\, d^3z\, \langle \overline{\psi}(x_0)\gamma_0\gamma_5\tau^a\psi(x_0)\overline{\zeta}(y)\gamma_5\tau^a\zeta(z)\rangle$$

$$f_P(x_0) = -1/12 \int d^3y\, d^3z\, \langle \overline{\psi}(x_0)\gamma_5\tau^a\psi(x_0)\overline{\zeta}(y)\gamma_5\tau^a\zeta(z)\rangle$$

PCAC Mass Finite Size Effects



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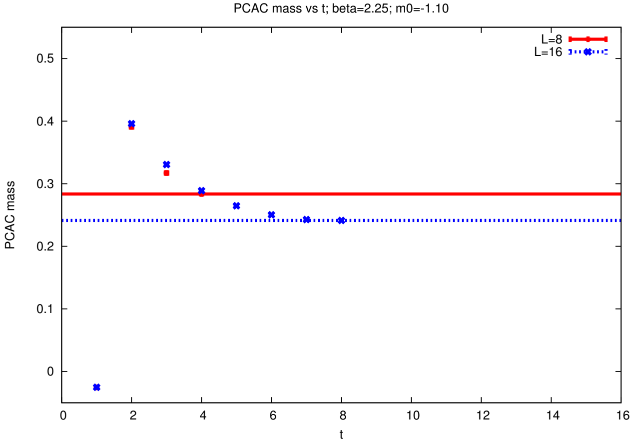
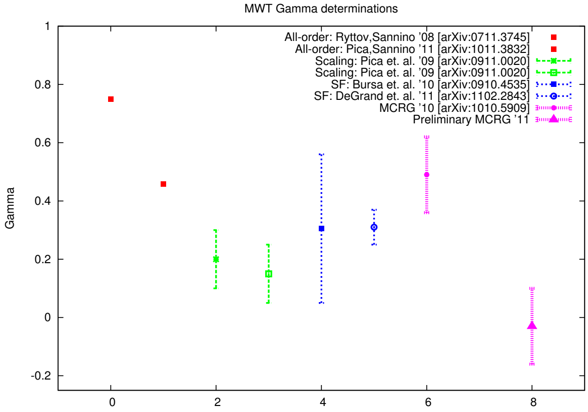


Results in Context



- We find $\gamma = -0.03(13)$
- $\gamma = 1$ is strongly excluded

Prediction for anomalous dimension

Conjectured all orders beta function

$$\beta(g) = \frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3} T(r) N_f \gamma(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta_0'}{\beta_0}\right)}$$

$$\beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} T(r) N_f , \quad \beta_0' = C_2(G) - T(r) N_f$$

- For MWTC this predicts anomalous dimension $\gamma = 3/4$ at fixed point
- This is a scheme-independent quantity at a fixed point

Ryttov, Sannino [arXiv:0711.3745]

Particle content of MWT

- Fermionic content:
  - (U,D) techni-quark doublet
  - (N,E) new lepton doublet
  - Composite techniquark-technigluon doublet
- Composite Higgs from techni-pion

MWT LHC Phenomenology

- Details depend on choice of ETC model
- Then construct low energy EFT for LHC

Frandsen, Sannino, et. al. [arXiv:0710.4333v1] [arXiv:0809.0793v1]

MWT Dark Matter candidate

- Lightest technibaryon is a cold dark matter candidate
- TIMP: Technicolour Interacting Massive Particle
- iTIMP: lightest weak isotriplet technibaryon
- Prospects for discovery/exclusion from both dark matter experiments and LHC

Frandsen, Sannino [arXiv:0911.1570]