Mass Anomalous Dimension at Large N

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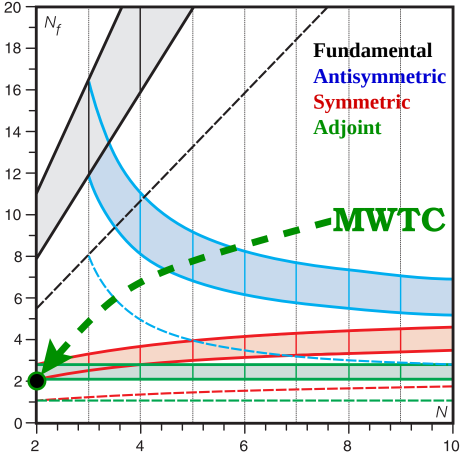
Dynamical Electroweak Symmetry Breaking



[arXiv:hep-ph/0611341]

- Dynamical EWSB or Technicolor Models
- In particular MWT: 2 dirac fermions transforming under the adjoint representation of SU(2)

Saninno, Tuominen [arXiv:hep-ph/0405209]

Mass Anomalous Dimension

Size of quark mass terms in the effective action depend on the value of the anomalous mass dimension γ.

Quark Masses

$$\frac{\langle\overline{\Psi}\Psi\rangle_{ETC}}{\Lambda^2_{ETC}}\overline{\psi}\psi$$

Power Enhancement

$$\langle\overline{\Psi}\Psi\rangle_{ETC} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma}\langle\overline{\Psi}\Psi\rangle_{TC}$$

- Need $\gamma \simeq 1$ to generate large enough quark masses.
- Important quantity to measure in TC models.

Method

At small eigenvalues, at leading order,

Spectral density of the Dirac Operator

$$\rho(\omega) \propto \mu^{\frac{4\gamma_*}{1+\gamma_*}} \omega^{\frac{3-\gamma_*}{1+\gamma_*}} + \dots$$

- Integral of this is the mode number, which is just counting the number of eigenvalues of the Dirac Operator on the lattice.
- Fitting this to the above form can give a precise value for γ, as done recently for MWT by Agostino Patella.

Patella [arXiv:1204.4432]

Why Large N?

- In perturbation theory, γ_* is independent of N, so we expect the large N value to be close to the $N = 2$ value.
- At large N the theory is (under certain conditions) volume independent, so the calculation can be done on a small lattice or even a single site.
- Interesting cross check of method, perturbation theory and large N volume independence.

Large–N Volume Independence

Eguchi-Kawai '82

In the limit $N_c \to \infty$, the properties of U(N_c) Yang–Mills theory on a periodic lattice are independent of the lattice size.

$$S_{YM} = S_{EK} \equiv \frac{N_c}{\lambda} \sum_{\mu<\nu} Tr \left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

where $\lambda \equiv g^2 N_c$ is the bare 't Hooft coupling, held fixed as $N_c \to \infty$.

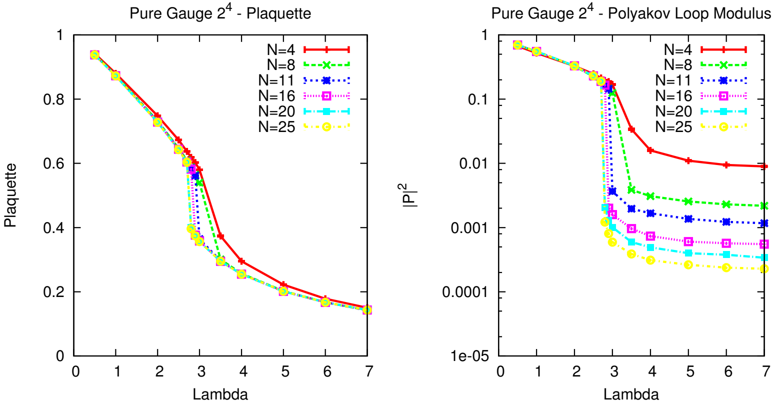
Conditions

...but it turns out only

- for single–trace observables defined on the original lattice of side L, that are invariant under translations through multiples of the reduced lattice size L'
- and if the $U(1)^d$ center symmetry is not spontaneously broken, i.e. on the lattice the trace of the Polyakov loop vanishes.

Pure Gauge Phase Diagram

Pure Gauge: Plaquette and Polyakov loop vs λ.



Twisted Eguchi–Kawai

Gonzalez–Arroyo Okawa '83

Impose twisted boundary conditions, such that the classical minimum of the action preserves a Z_N^2 subgroup of the center symmetry.

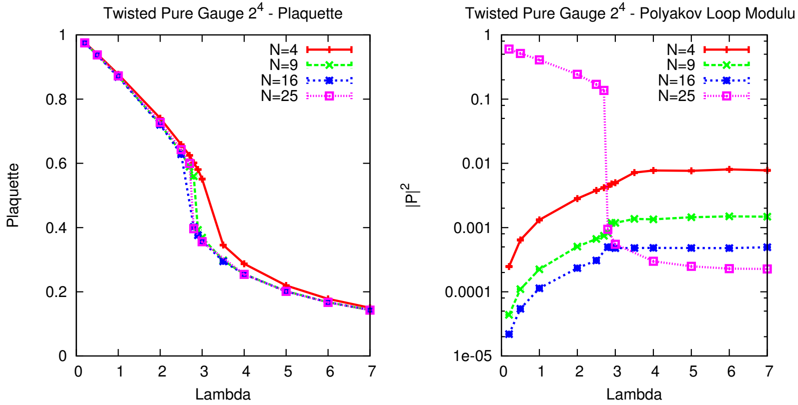
$$S_{TEK} = \frac{N_c}{\lambda} \sum_{\mu<\nu} Tr \left(z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

$$\text{where } z_{\mu\nu} = exp\{2\pi ik/\sqrt{N}\}$$

- Original choice is $k = 1$

Twisted Pure Gauge Phase Diagram

$k = 1$ Twisted Pure Gauge: Plaquette and Polyakov loop vs λ.



Twisted Eguchi–Kawai

- Original choice $k = 1$ seen to break center–symmetry at intermediate couplings for $NL^2 \gtrsim 100$
- But symmetry can be restored by scaling the twist k with N

Gonzalez–Arroyo Okawa [arXiv:1005.1981]

- See the talks by Antonio Gonzalez-Arroyo and Masanori Okawa.

QCDadj

Kotvul Unsal Yaffe '07

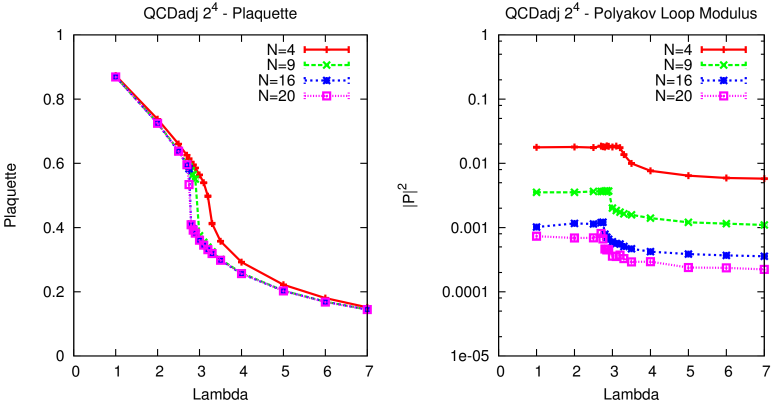
Add (massless or light) adjoint fermions with periodic boundary conditions

- Preserves center symmetry down to a single site
- and for a range of light adjoint fermion masses.

- Works in perturbation theory (for $am \lesssim \frac{1}{N}$)
- And in lattice simulations (possibly even for $am \lesssim 1$)

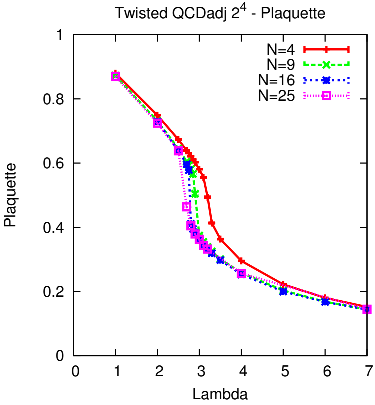
QCDadj Phase Diagram

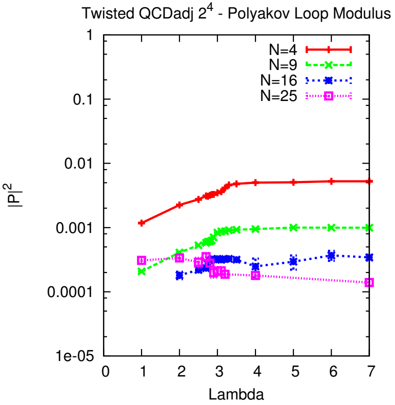
QCDadj ($am_0 = 0$): Plaquette and Polyakov loop vs λ.



QCDadj+Twist Phase Diagram

QCDadj+Twist ($am_0 = 0$): Plaquette and Polyakov loop vs λ.



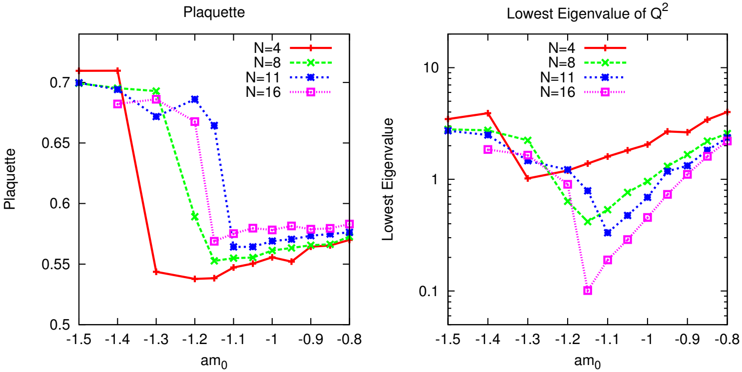


Simulation Details

- Simulate QCDadj.
  - SU(N) gauge theory with 2 adjoint Dirac fermions with periodic boundary conditions.
- Use 2^4 lattices with N up to 20.
  - $V_{eff} \sim L^4 N^2$, so equivalent to $N \sim 80$ on a single site.
- Measure lowest 5% of eigenvalues of the Dirac operator Q^2.
  - Scales with N^2, ~ 400 eigenvalues for $N = 16$.
- Choose initial bare coupling $\lambda = 2.80$.
  - Need to stay in weak coupling phase.
  - But want fairly strong coupling to minimise $1/N$ effects.
- Simulate with and without the minimal symmetric twist.

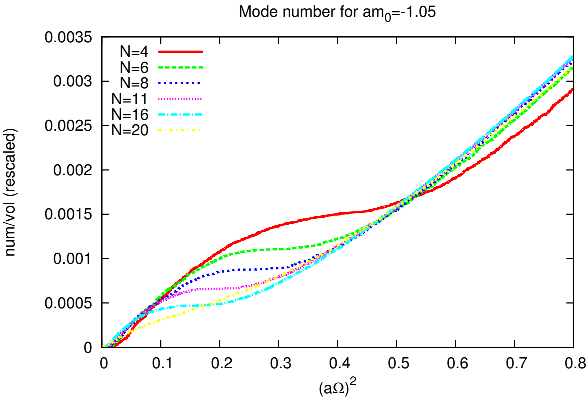
QCDadj: Bare Coupling and Critical Bare Mass

QCDadj at $\lambda = 2.80$: Scan of the plaquette and lowest Dirac operator eigenvalue as a function of the bare mass.
Want as light a mass as possible, while avoiding the Aoki phase.



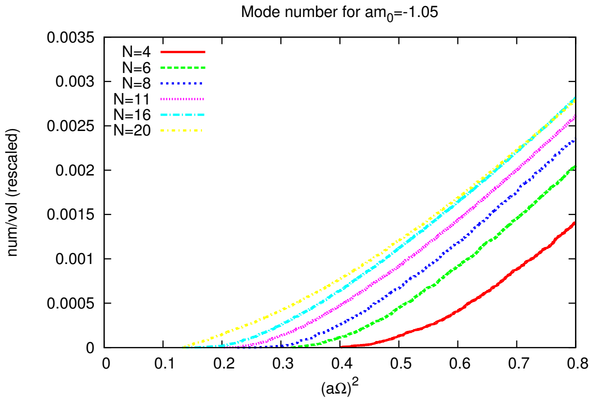
QCDadj Mode Number - Zero modes

Finite volume effect: $4(N-1)$ would–be zero modes.
These are suppressed by the volume $(1/N^2)$ in the large–N limit.



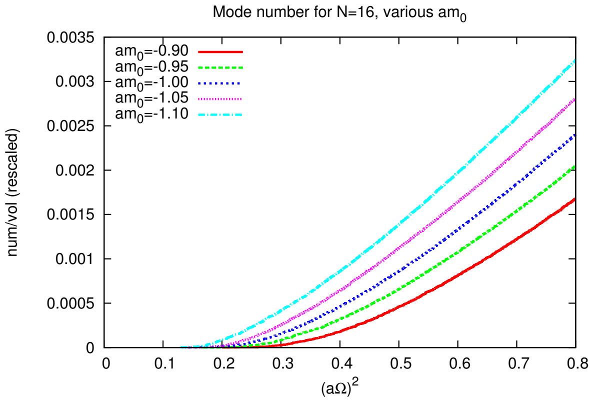
QCDadj Mode Number - N dependence

Mass $am_0 = -1.05$, Dirac mode number for various N (zero modes subtracted).



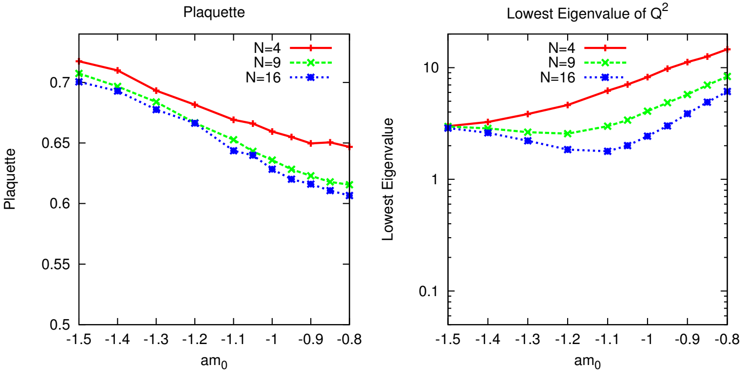
QCDadj Mode Number - Mass dependence

N=16, Dirac mode number for various masses am_0 (zero–modes subtracted).



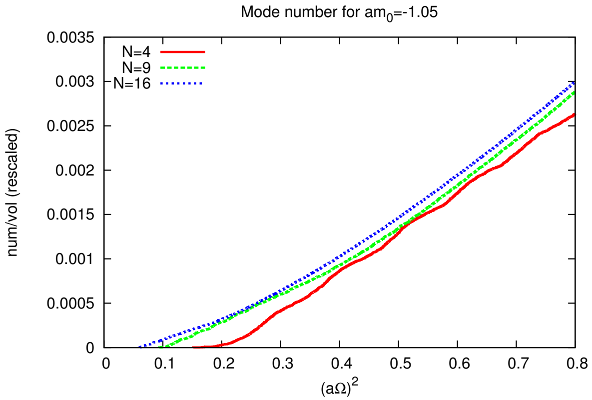
QCDadj+Twist: Bare Coupling and Critical Bare Mass

QCDadj+Twist at $\lambda = 2.80$; the lowest eigenvalue of the Dirac operator gives a similar critical bare mass, but do not see a discontinuity in the plaquette.



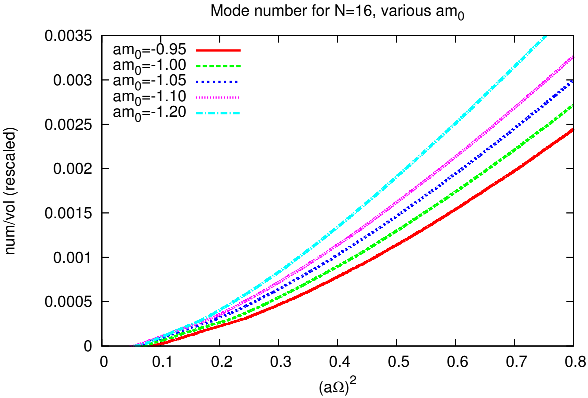
QCDadj+Twist Mode Number - N dependence

At $am_0 = -1.05$, do not see the would–be zero modes, they are suppressed by the twist. Otherwise similar to the untwisted case.



QCDadj+Twist Mode Number - Mass dependence

N=16 mass–dependence.



Method

Fit data to the function

$$a^{-4}\overline{\nu}(\Omega) \simeq a^{-4}\overline{\nu}_0 + A\left[(a\Omega)^2 - (am)^2\right]^{\frac{2}{1+\gamma_*}}$$

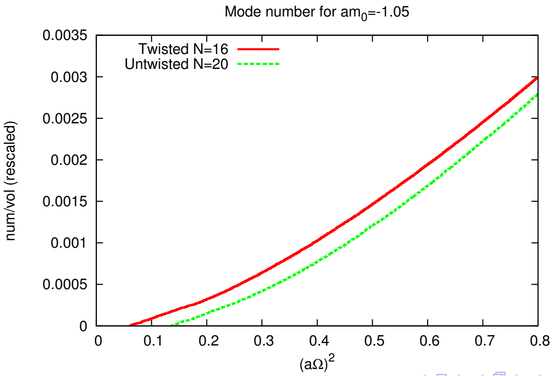
in some intermediate range $a\Omega_L < a\Omega < a\Omega_H$ where

- $a^{-4}\overline{\nu}(\Omega)$ is the number of eigenvalues of Q^2 below Ω^2 divided by the volume
- $a^{-4}\overline{\nu}_0$ is a fitted parameter (contribution of small excluded eigenvalues, $\propto M_{PS}^4$)
- am is a fitted parameter (physical mass)
- A is a fitted parameter

Patella [arXiv:1204.4432]

Anomalous Mass Dimension Fit

Example fit to $A\left[(a\Omega)^2 - (am)^2\right]^{\frac{2}{1+\gamma_*}}$ for $0.2 < (a\Omega)^2 < 0.8$
(Only illustrative: many free parameters, and unknown systematics from finite mass and volume effects at this stage)



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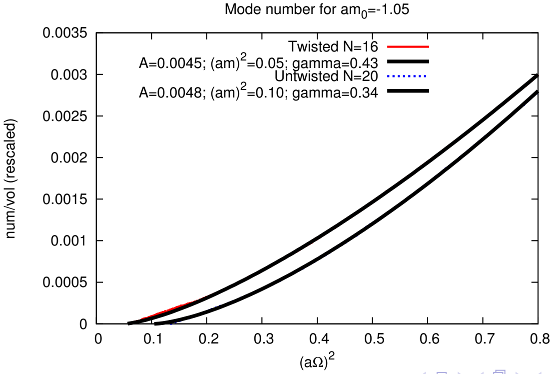
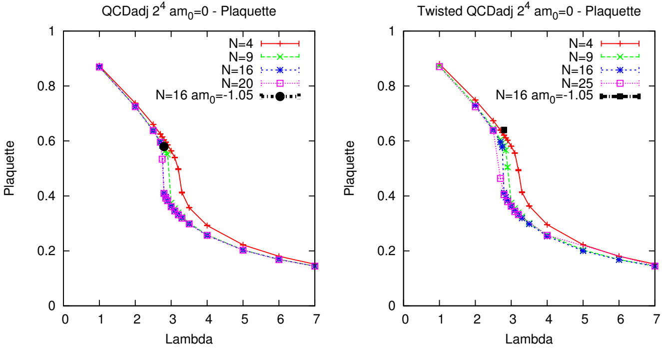


Conclusion

- Promising initial results.
  - Volume reduction seems to work.
  - Eigenvalue spectrum looks sensible.
- Now need to go to larger N and lighter masses.
- Also need to investigate the systematics of the fitting procedure.
- And want to try different twist and couplings.

Correct Phase

The value of the plaquette for $\lambda = 2.80$ at $am_0 = -1.05$ is consistent with being in the weak coupling phase.



Strong Coupling Phase

In the (unphysical) strong coupling phase, no zero modes, $1/N$ corrections are tiny, and quenched configurations, with or without twist, give the same Dirac operator spectrum as dynamical ones.