TEK twisted gradient flow running coupling

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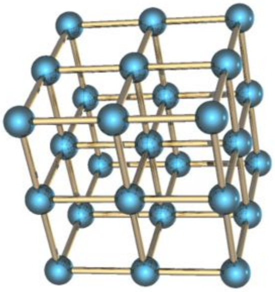
Lattice Field Theory



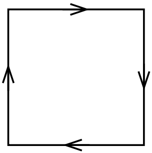
Formulate field theory on a discrete set of space–time points:

- $\hat{L}^4$ points, lattice spacing a
- Physical volume $L^4 = (\hat{L}a)^4$

Lattice provides regularisation:

- UV cut–off: $1/a$
- IR cut–off: $1/L$

Lattice Field Theory



The simplest lattice discretisation of the Yang–Mills action is

$$S_{YM} = N_c b \sum_x \sum_{\mu<\nu} Tr \left(U_\mu(x) U_\nu(x+\mu) U_\mu^\dagger(x+\nu) U_\nu^\dagger(x) + h.c. \right)$$

where $b = \frac{1}{\lambda} = \frac{1}{g^2 N_c}$ is the inverse bare 't Hooft coupling, held fixed as $N_c \to \infty$.

Large–N Volume Independence

Eguchi-Kawai '82

In the limit $N_c \to \infty$, the properties of U(N_c) Yang–Mills theory on a periodic lattice are independent of the lattice size.

$$S_{YM} \equiv S_{EK} = N_c b \sum_{\mu<\nu} Tr \left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

where $b = \frac{1}{\lambda} = \frac{1}{g^2 N_c}$ is the inverse bare 't Hooft coupling, held fixed as $N_c \to \infty$.

Conditions

...but it turns out only

- for single–trace observables defined on the original lattice of side L, that are invariant under translations through multiples of the reduced lattice size L'
- and if the $U(1)^d$ center symmetry is not spontaneously broken, i.e. on the lattice the trace of the Polyakov loop vanishes.

Twisted Reduction

Gonzalez–Arroyo Okawa '83

Impose twisted boundary conditions, such that the classical minimum of the action preserves a Z_N^2 subgroup of the center symmetry.

$$S_{TEK} = N_c b \sum_{\mu<\nu} Tr \left(z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

$$z_{\mu\nu} = exp\{2\pi i n_{\mu\nu}/N\} = z_{\nu\mu}^*$$

Gonzalez–Arroyo Okawa [arXiv:1005.1981]

Twisted Reduction

Choice of flux k

$$n_{\mu\nu} = k\sqrt{N}, \quad k\bar{k} = 1 \mod \sqrt{N}, \quad \tilde{\theta} = 2\pi\bar{k}/\sqrt{N}$$

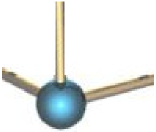
Original TEK: $k = 1$, center-symmetry breaks for $N \gtrsim 100$

To take $1/N \to 0$ limit, choose k such that

- $k/\sqrt{N} > 1/9$
- $\tilde{\theta} =$ constant

Garcia–Perez Gonzalez–Arroyo Okawa [arXiv:1307.5254]

Twisted Reduction



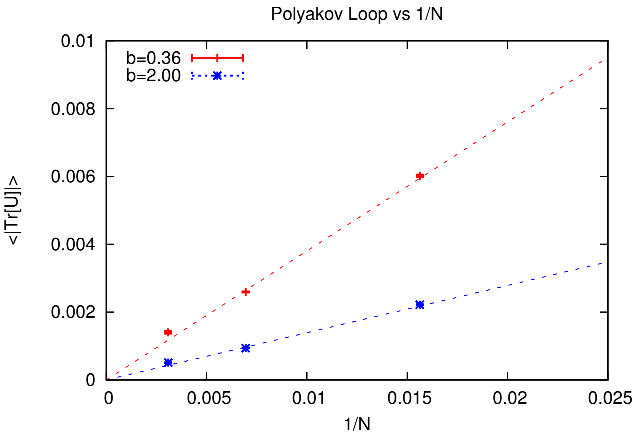
Twisted reduction: $\hat{L} \to \sqrt{N}$

- Single site lattice, lattice spacing a
- Physical volume $L^4 = (\sqrt{N}a)^4$

Lattice provides regularisation:

- UV cut–off: $1/a$
- IR cut–off: $1/\sqrt{N}a$

Polyakov Loop vs 1/N



Wilson Flow

The Wilson flow evolves the gauge field according to

Flow Equation

$$\frac{\partial B_\mu}{\partial t} = D_\nu G_{\nu\mu}, \qquad B_\mu\big|_{t=0} = A_\mu$$

where A_μ is the gauge field, and t is the flow time.
This integrates out UV fluctuations above a scale $\mu = 1/\sqrt{8t}$ (i.e. smears observables over a radius $\sqrt{8t}$)

Lüscher [arXiv:0907.5491]

Wilson Flow of $\frac{1}{N}t^2\langle E\rangle$

The action density $E = G_{\mu\nu}G_{\mu\nu}$ as a function of flow time can be used to define a scale t_0

Definition of scale t_0

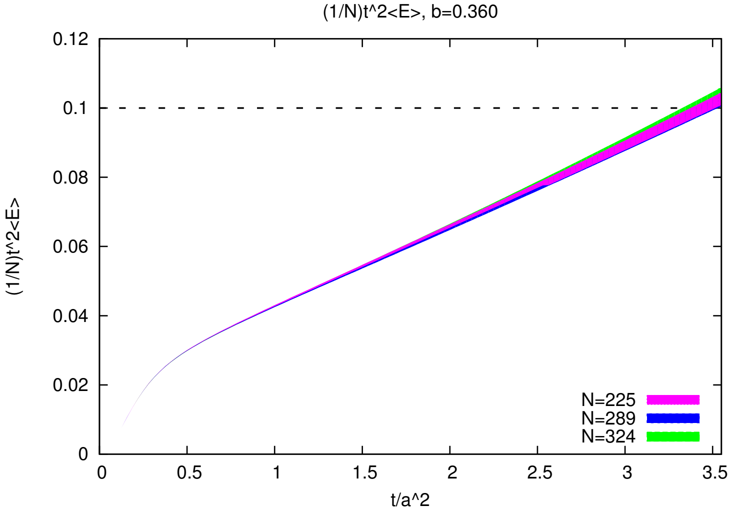
$$\frac{1}{N}t_0^2\langle E(t_0)\rangle = 0.1$$

Perturbative expansion of E at small flow time t

$$\frac{1}{N}t^2E(t) = \frac{3\lambda}{128\pi^2}\left[1 + \frac{\lambda}{16\pi^2}(11\gamma_E/3 + 52/9 - 3\ln 3)\right]$$

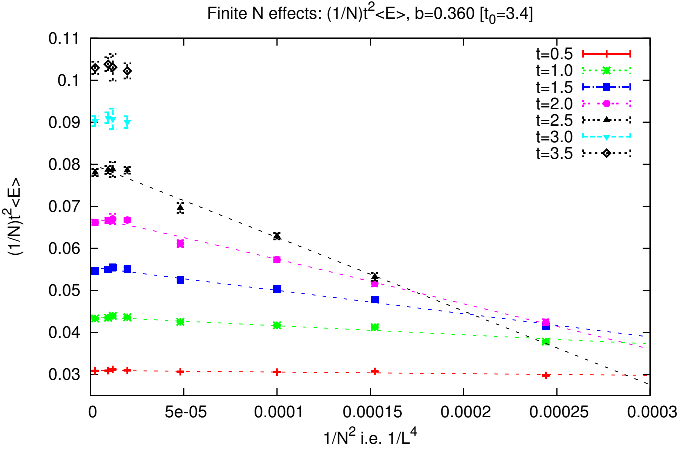
Lüscher [arXiv:1006.4518]

Setting the scale with t_0

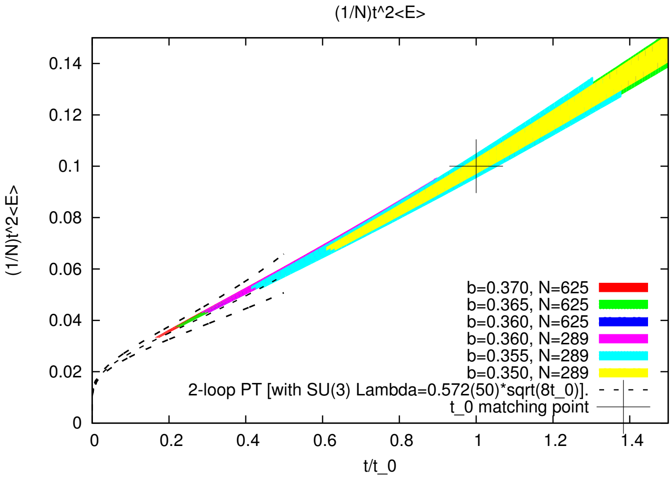


Finite Volume Effects of $\frac{1}{N} t^2 \langle E \rangle$

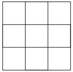
Finite volume effects $\propto L^{-4}$ i.e. N^{-2} Lüscher [arXiv:1404.5930]



Comparison to SU(3) Perturbation Theory



Running of the coupling: Step Scaling



- Step scaling - change in coupling from L to sL
- $u = \overline{g}^2(b, a/L, L)$
- $\Sigma(u, s, a/L) = \overline{g}^2(b, a/L, sL)$
- $\sigma(u, s) = \lim_{a/L \to 0} \Sigma(u, s, a/L)$
- Now tune bare parameters until $\overline{g}^2(b, a/L, L) = \sigma(u, s)$
- Repeat

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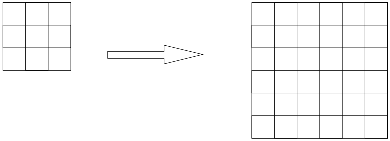

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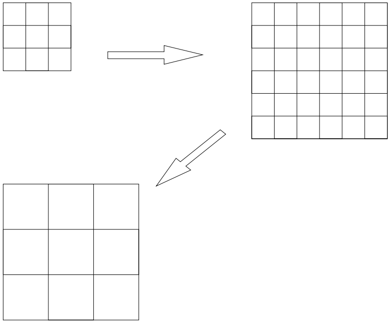

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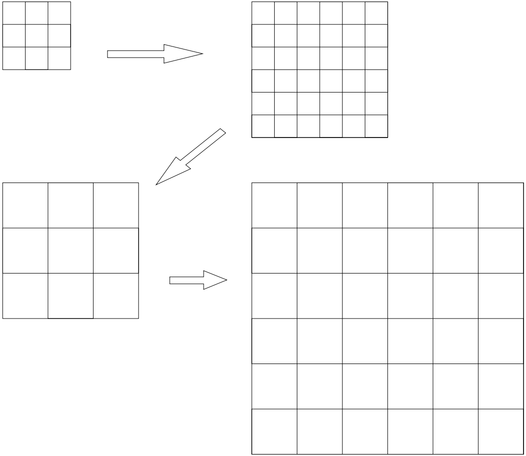

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Twisted Gradient Flow Scheme

Define a renormalised coupling in terms of E at positive flow time:

Definition of renormalised coupling λ_{TGF}

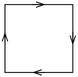
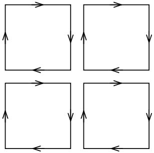
$$\lambda_{TGF}(L) = \mathcal{N}_T^{-1}(c) t^2 \langle E \rangle \Big|_{t=c^2N/8} = \lambda_{\overline{\text{MS}}} + \mathcal{O}(\lambda_{\overline{\text{MS}}}^2)$$

- Smearing radius is a fraction c of the lattice size $\sqrt{8t} = cL = c\sqrt{N}a$
- Renormalisation scale is the inverse of the box size $\mu = 1/L$.
- c is a free parameter, defines renormalisation scheme.

Ramos [arXiv:1308.4558]

Lattice Discretisation Effects

Need to choose a discretisation for E:

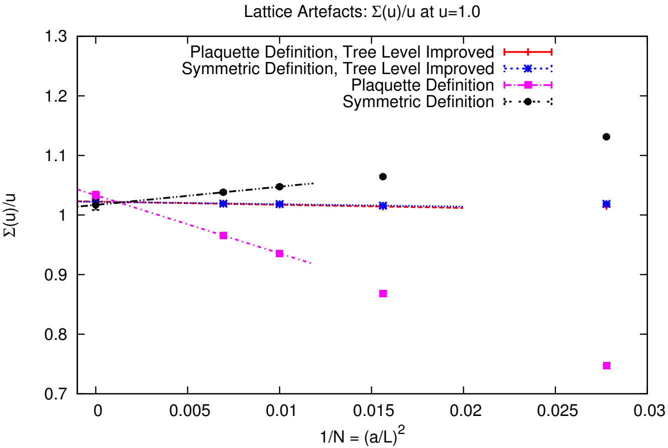
- Plaquette  or Symmetric 

Also have a choice for $\mathcal{N}_T$:

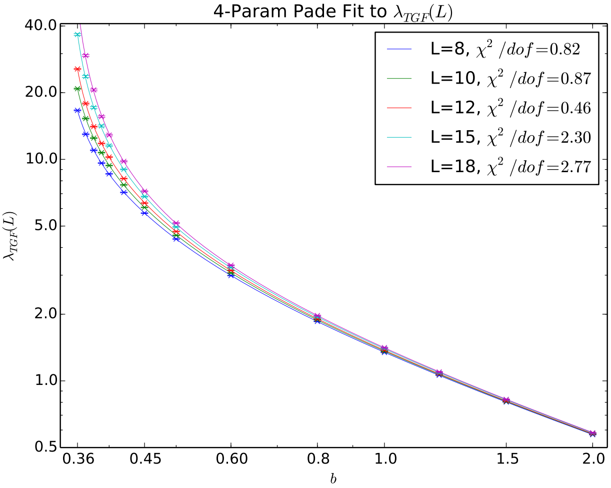
- Tree level continuum definition
- Tree level lattice definition

All equivalent up to $\mathcal{O}(a/L)^2$ lattice artefacts.

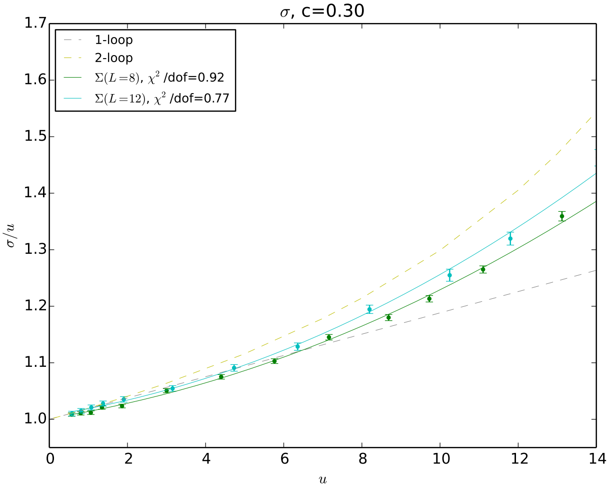
Lattice Artefacts, $u = 1, c = 0.30$



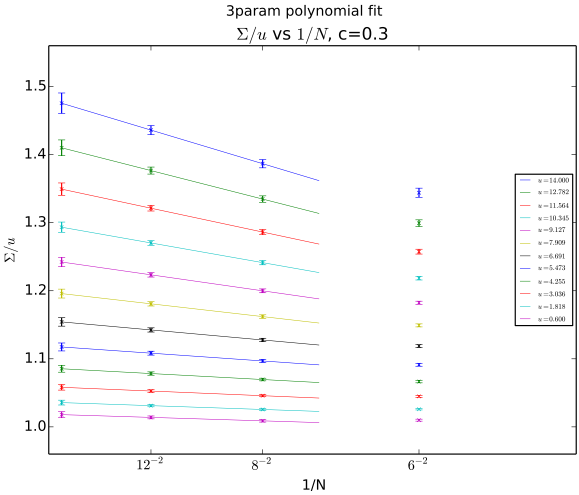
Twisted Gradient Flow Coupling for $c = 0.30$



Lattice Discrete Beta Function [preliminary]



Continuum Extrapolation [preliminary]



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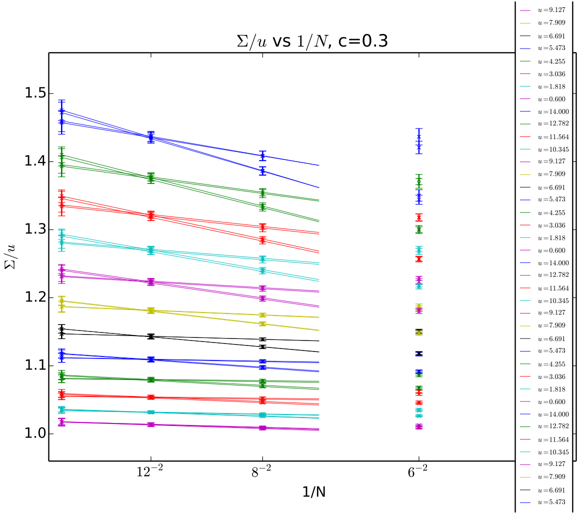

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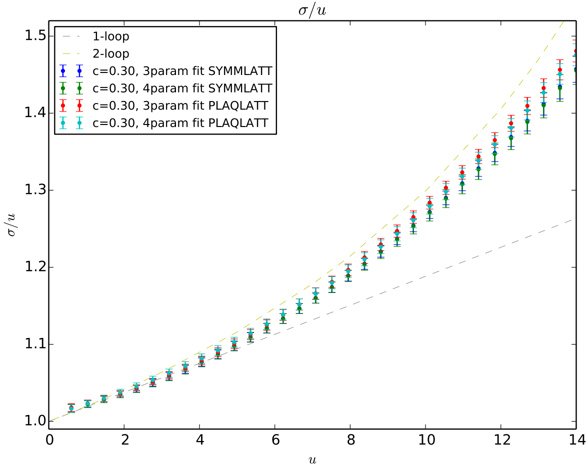


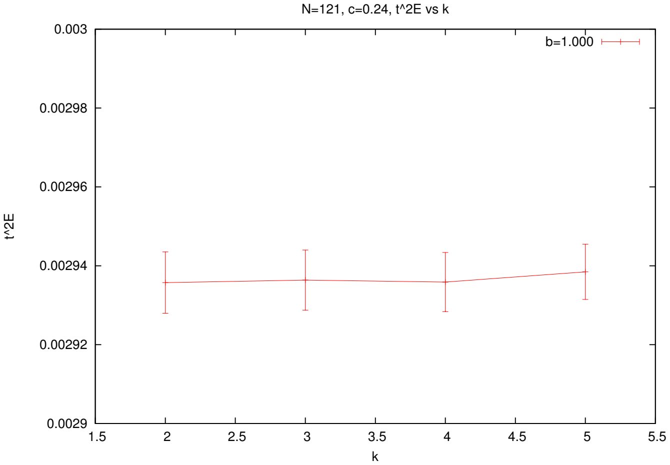
Conclusion and Future Work

- Promising initial results.
    - Twisted volume reduction seems to work
    - Good agreement with perturbation theory

Future Work:

- Better understanding of systematic errors
- $n_f = 2$ running coupling study

Theta dependence - N=121,b=1.00



Theta dependence - N=121,b=0.360