Schwinger Model Mass Anomalous Dimension

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Talk Outline

- Hard problem: Investigating IR conformal theories on the lattice
- One approach: Determine Mass Anomalous Dimension from Dirac Operator Mode Number
- Systematics: To understand the systematic errors, apply method to a well understood toy model.
- Toy model: Massive $n_f = 2$ Schwinger Model

Mode Number Method

In a mCFT, the condensate goes like

Chiral Condensate

$$\lim_{m\to 0}\lim_{V\to\infty}\langle\bar{\psi}\psi\rangle \propto m^{\frac{d}{1+\gamma_*}-1}+\ldots$$

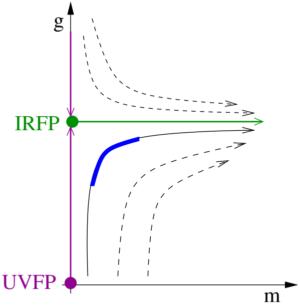
and in the infinite volume, chiral limit, and for small eigenvalues ω,

Spectral density of the Dirac Operator

$$\lim_{m\to 0}\lim_{V\to\infty}\rho(\omega) \propto \omega^{\frac{d}{1+\gamma_*}-1}+\ldots$$

DeGrand [arXiv:0906.4543], Del Debbio et. al. [arXiv:1005.2371], Patella [arXiv:1204.4432], Hasenfratz et. al. [arXiv:1303.7129]

Mode Number Fit Range



RG flows in mass–deformed CFT:

- Flow from UV (high eigenvalues) to IR (low eigenvalues)
- Finite mass drives us away from FP in the IR
- Interested in intermediate blue region
- $\frac{1}{L} \ll m \ll \Omega_{IR} < \Omega < \Omega_{UV} \ll \frac{1}{a}$

Fit Function I

Split low and high eigenvalue contributions to the mode number:

$$\nu(\Omega) = \int_0^{\sqrt{\Omega_{IR}^2 - m^2}} \rho(\omega) d\omega + \int_{\sqrt{\Omega_{IR}^2 - m^2}}^{\sqrt{\Omega^2 - m^2}} \rho(\omega) d\omega$$

Inserting $\rho(\omega) \sim \omega^{\frac{d}{1+\gamma_*} - 1}$ in the second term:

Mode number fit function

$$\nu(\Omega) = \nu(\Omega_{IR}) + A \left[\left(\Omega^2 - m^2\right)^{\frac{d/2}{1+\gamma_*}} - \left(\Omega_{IR}^2 - m^2\right)^{\frac{d/2}{1+\gamma_*}} \right]$$

- Fit in range $\Omega_{IR} < \Omega < \Omega_{UV}$.
- 3 fit parameters: A, m and γ_*.

Fit Function II

If finite mass and finite volume effects are negligible:

$$m^2 \simeq 0, \quad \nu(\Omega_{IR}) \simeq 0$$

Mode number fit function

$$\nu(\Omega) = A\Omega^{\frac{d}{1+\gamma_*}}$$

- Fit in range $\Omega_{IR} < \Omega < \Omega_{UV}$.
- 2 fit parameters: A, and γ_*.

Fit Function III

Alternatively can bin the measured eigenvalues and fit to the spectral density,

Spectral density fit function

$$\rho(\Omega) = A\Omega^{\frac{d}{1+\gamma_*}-1}$$

- Fit in range $\Omega_{IR} < \Omega < \Omega_{UV}$.
- 2 fit parameters: A, and γ_*.

Massive Schwinger Model

Schwinger Model Lagrangian

$$\overline{\psi}_f(x)\left[\gamma_\mu(i\partial_\mu + gA_\mu(x)) + m\right]\psi_f(x) + \frac{1}{2}F_{\mu\nu}(x)F_{\mu\nu}(x)$$

- 2 dimensional QED, i.e. U(1) gauge field.
- n_f massive Dirac fermions

Chiral Condensate

$$\lim_{m\to 0}\lim_{V\to\infty}\langle\bar{\psi}\psi\rangle \propto m^{(n_f-1)/(n_f+1)}$$

Smilga 1992, Hetrick et. al. 1995

Mass Anomalous Dimension

Chiral Condensate

$$\lim_{m\to 0} \lim_{V\to\infty} \langle \bar{\psi}\psi \rangle \propto m^{\frac{d}{1+\gamma_*}-1} + \dots$$

In the language of mCFT, known UV and IR limits:

IR and UV limits

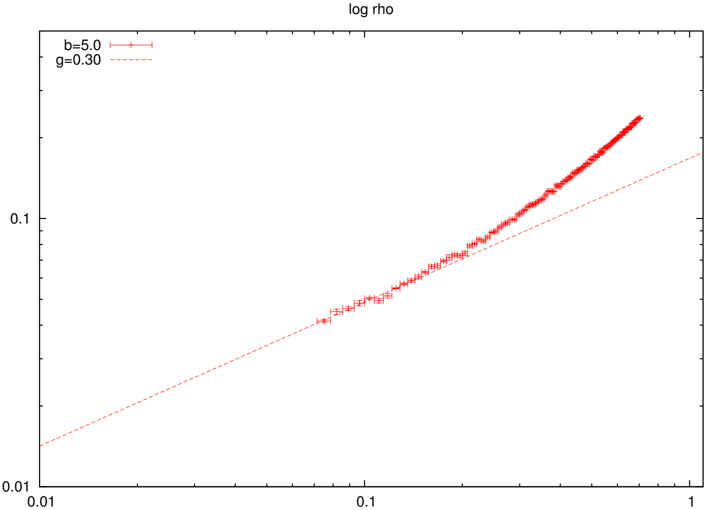
$$\gamma_* = \begin{cases} 0 & \text{for} \quad mL\sqrt{\mu L} \ll 1 \quad (UVFP) \\ 0.5 & \text{for} \quad mL\sqrt{\mu L} \gg 1 \quad (IRFP) \end{cases}$$

Smilga 1992, Hetrick et. al. 1995
Bietenholz et. al. [arXiv:1109.2649 [hep-lat]]

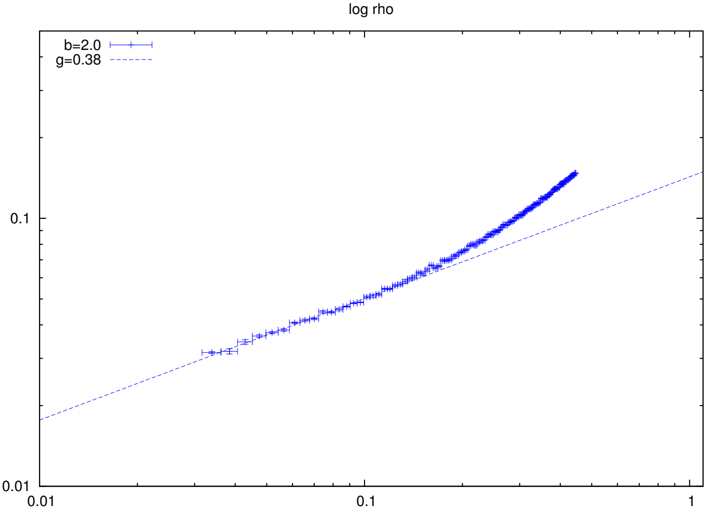
Simulation Details

- Unimproved Wilson Fermions
- Wilson gauge action (compact gauge links)
- Lattice Volumes: 16^2, 24^2, 32^2
- Lattice Spacing: $\beta = 0.1, 0.5, 1.0, 2.0, 5.0$
- Eigenvalues: measure lowest $\sim 15\%$, e.g. lowest 300 eigenvalues on 32^2 lattice.

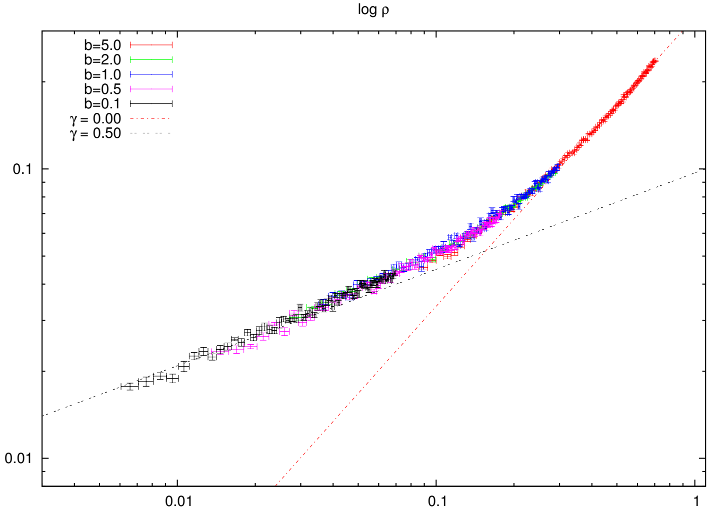
Spectral Density



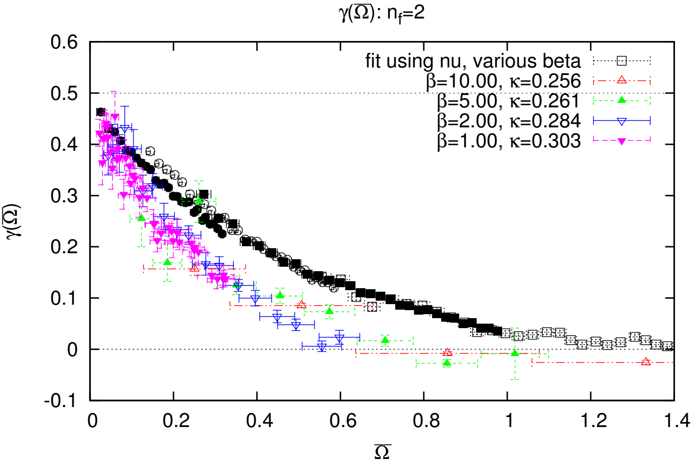
Spectral Density



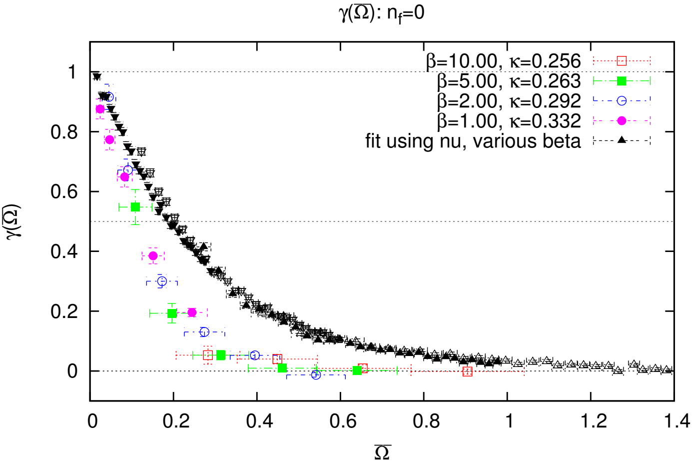
Spectral Density



Mass Anomalous Dimension $n_f = 2$



Mass Anomalous Dimension $n_f = 0$



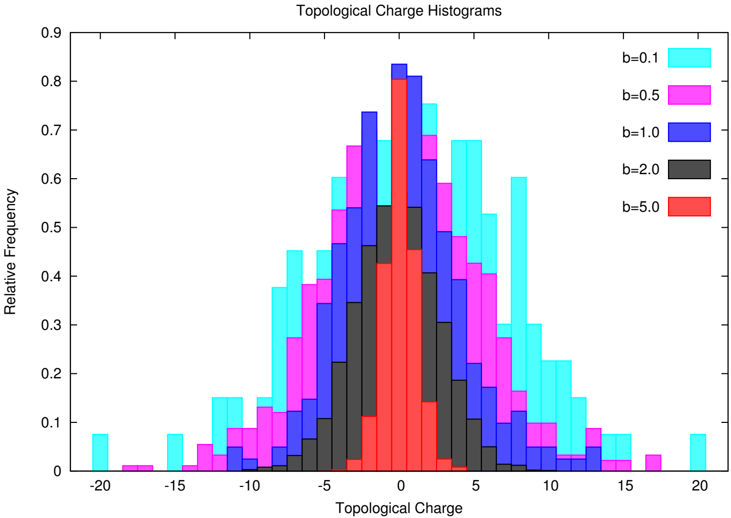
Conclusion and Future Work

- Method works
  - Approaches correct value in IR limit.
  - But hard to say a priori what the systematic error is without knowing the correct answer already.

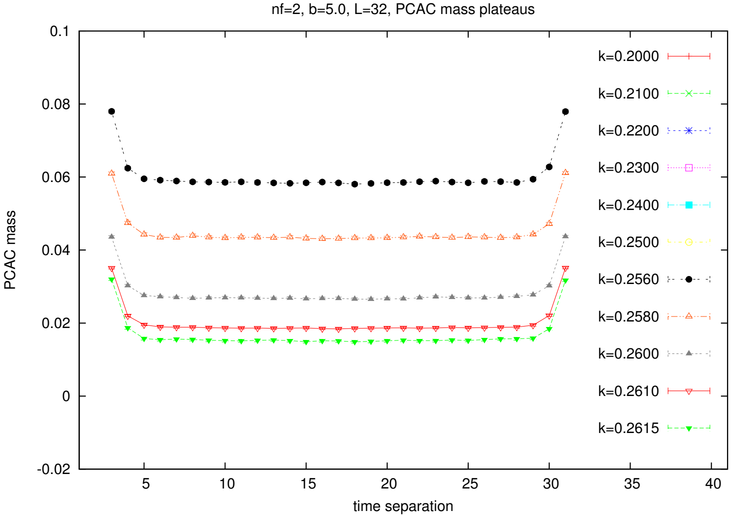
Future Work:

- Continuum limit, IR extrapolation
- Massless simulations with SF boundary conditions?

Topological Charge Histograms



PCAC Mass Plateaus



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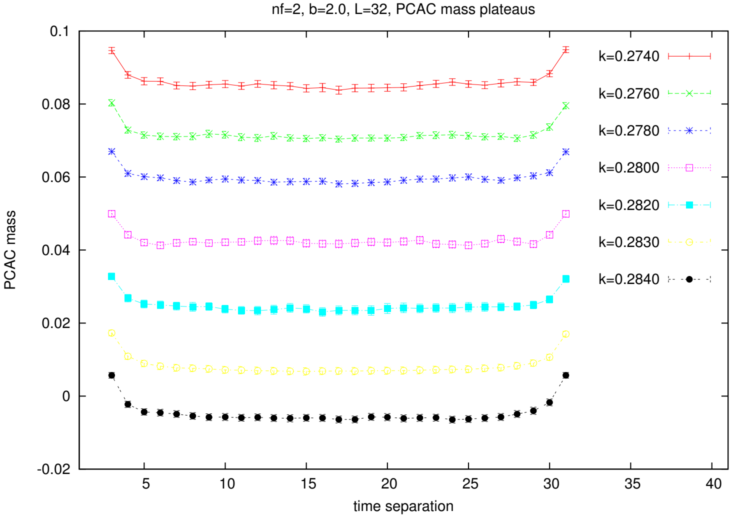

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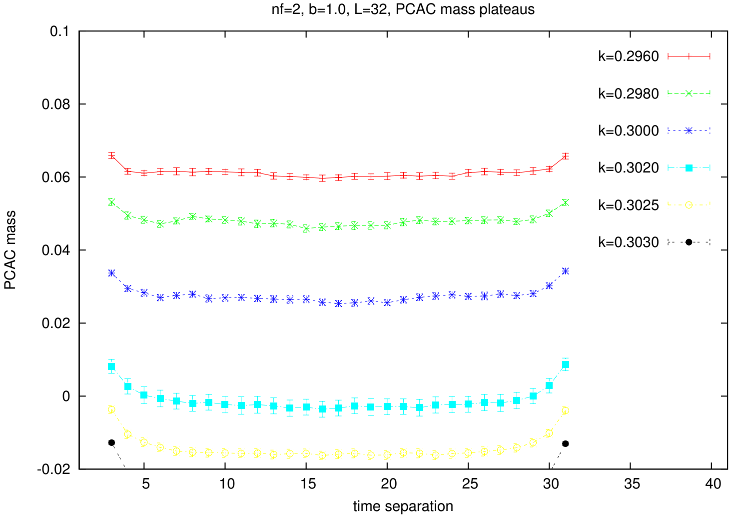

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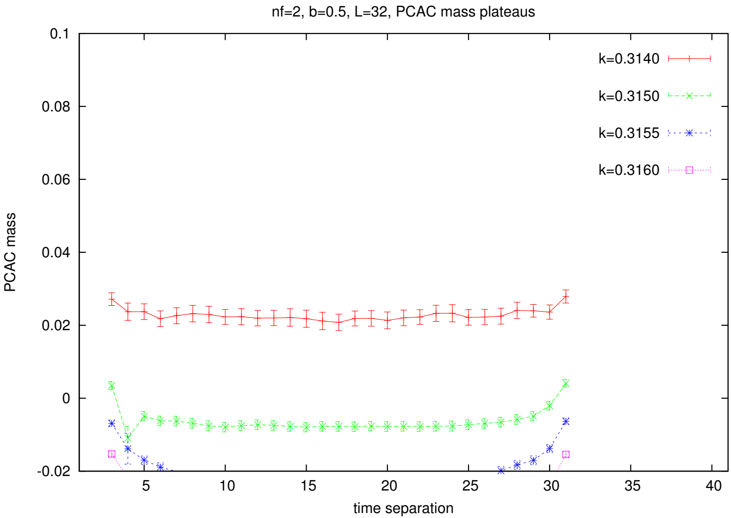

PCAC Mass Plateaus