Mass Anomalous Dimension at Large N

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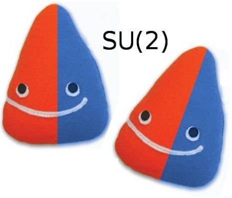
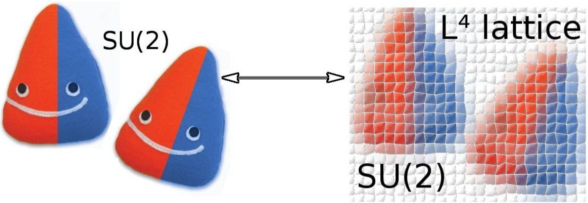
IFT UAM/CSIC, Universidad Autónoma de Madrid, Spain.

Outline of Talk



Outline of Talk



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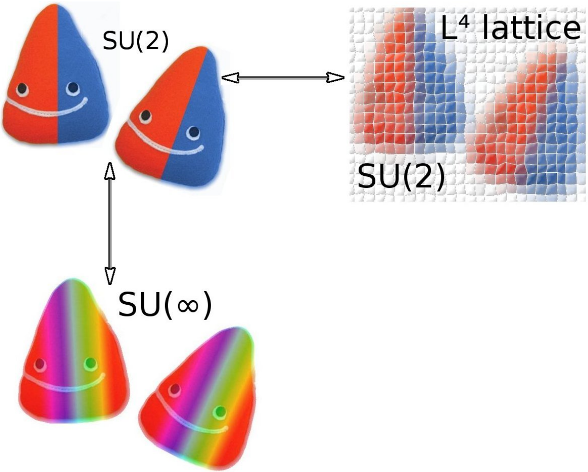

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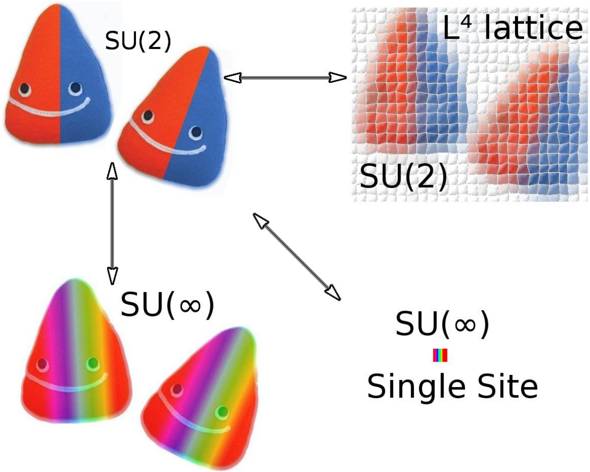
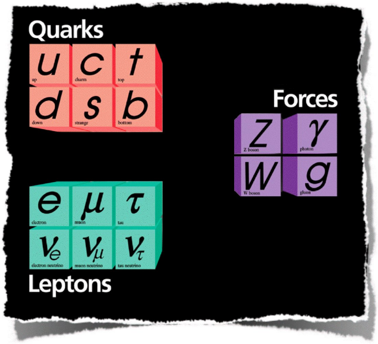


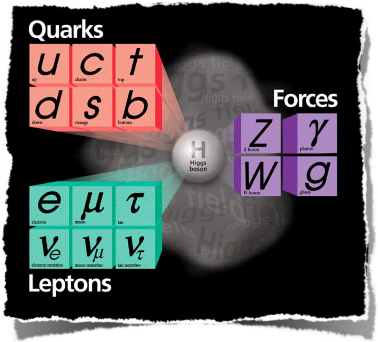
The Standard Model



Fermilab

- Standard Model is well verified experimentally
- Electroweak Symmetry breaking included (i.e. mass of W/Z bosons)
- And finally also a Higgs Boson of mass $\sim$ 125 GeV.

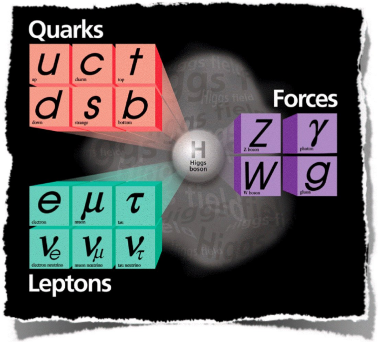
The Standard Model



Fermilab

- Standard Model is well verified experimentally
- Electroweak Symmetry breaking included (i.e. mass of W/Z bosons)
- And finally also a Higgs Boson of mass $\sim$ 125 GeV.

The Higgs Mechanism



Fermilab

- SM Higgs has a few theoretical issues
  - Ad hoc: all fermion masses and mixings arbitrary parameters
  - Trivial: without new physics, Higgs decouples
  - Unnatural: quadratically sensitive to Planck scale, so requires fine tuning
- So thought to be an effective description of a more fundamental theory, e.g. SUSY, Technicolor, ...

Technicolor

- SM without Higgs already has some EW symmetry breaking.
- Quark condensate gives M_W of the order of the pion decay constant:

$$\langle \overline{u}_L u_R + \overline{d}_L d_R \rangle \neq 0 \rightarrow M_W = \frac{gF_\pi}{2} \sim 30 MeV$$

- So why not have some more 'techni-quarks' that form a condensate at a higher scale ($F_\pi^{TC} \sim 250 GeV \sim \Lambda_{TC}$)

Weinberg 78, Susskind 78

Extended Technicolor

- Add interactions between SM quarks and techni-quarks at some high scale Λ_{ETC}
- Get SM quark mass terms in effective low energy lagrangian:

Quark Masses

$$\frac{\langle\overline{\Psi}\Psi\rangle_{ETC}\overline{\psi}\psi}{\Lambda_{ETC}^2}$$

Dimopoulos, Susskind 79 - Eichten, Lane 80

Flavour Changing Neutral Currents

- But also get FCNC term:

Quark Masses

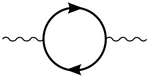
$$\frac{\langle\overline{\Psi}\Psi\rangle_{ETC}\overline{\psi}\psi}{\Lambda_{ETC}^2}$$

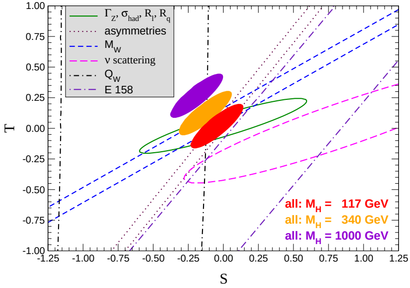
FCNC

$$\frac{\overline{\psi}\psi\overline{\psi}\psi}{\Lambda_{ETC}^2}$$

- Naively scaling up QCD leads to a problem:
- Need large $\Lambda_{ETC} \sim 1000\,TeV$ to suppress Flavour Changing Neutral Currents
- But this gives a strange quark mass that is ~ 50 times too small

S, T Parameters

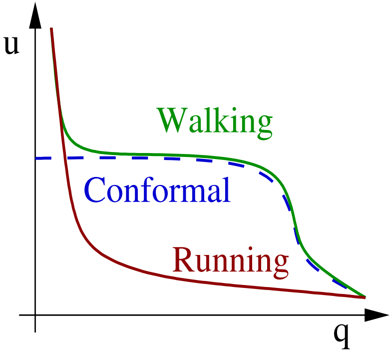


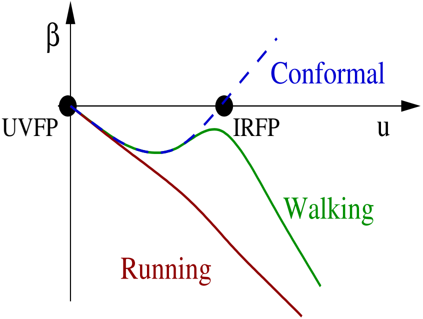


Particle Data Group 2008

- S,T parameters measure deviation from SM caused by new physics
- Naive QCD scaling gives $\sim 2\sigma$ disagreement with experiment
- Perturbative estimate: $S = \frac{1}{6\pi}\frac{N_f}{2}d(R) = 0.16$

Walking Technicolor Cartoon





Walking Technicolor Quark Masses

$$\langle\overline{\Psi}\Psi\rangle_{ETC} = \langle\overline{\Psi}\Psi\rangle_{TC} exp \left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \gamma(\mu) d \ln \mu \right)$$

- In QCD this gives logarithmic enhancement:

$$\langle\overline{\Psi}\Psi\rangle_{ETC} = \log\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma} \langle\overline{\Psi}\Psi\rangle_{TC}$$

- But a walking coupling gives power enhancement:

$$\langle\overline{\Psi}\Psi\rangle_{ETC} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma} \langle\overline{\Psi}\Psi\rangle_{TC}$$

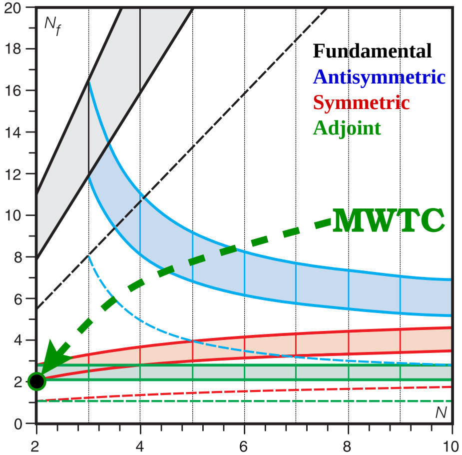
Walking Technicolor S Parameter

- Walking seems to reduce S parameter compared to running case.
- And other sectors of the theory, such as new leptons, are expected to contribute negatively

Dietrich, Sannino, Tuominen [arXiv:hep-ph/0505059]

- But ideally this also needs to be studied non-perturbatively

Phase Diagram



[arXiv:hep-ph/0611341]

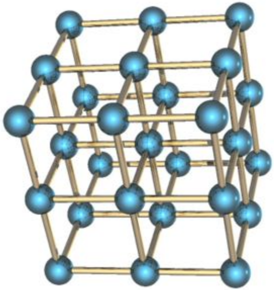
- MWTC: 2 dirac fermions transforming under the adjoint representation of SU(2)

Saninno, Tuominen [arXiv:hep-ph/0405209]

Scheme dependence

- Walking/Running of coupling is scheme dependent
- Want to measure physical, scheme independent quantities:
    - **Existence** of fixed point
    - **Anomalous mass dimension** at the fixed point

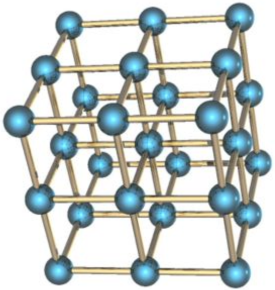
Lattice Field Theory



Formulate field theory on a discrete set of space–time points:

- Physical volume $L^4 = (Na)^4$
- N^4 points, lattice spacing a
- Quarks live on sites
- Gauge fields live on links between sites
- Simulate on a big computer

Continuum Limit



To recover continuum theory:

- UV cut–off: $1/a$
- IR cut–off: $1/L$
- Simulate for various $N = La$
- Then extrapolate to $a/L \to 0$

Large–N Volume Independence

Eguchi-Kawai '82

In the limit $N_c \to \infty$, the properties of U(N_c) Yang–Mills theory on a periodic lattice are independent of the lattice size.

$$S_{YM} = S_{EK} \equiv \frac{N_c}{\lambda} \sum_{\mu<\nu} Tr\left(U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c.\right)$$

where $\lambda \equiv g^2 N_c$ is the bare 't Hooft coupling, held fixed as $N_c \to \infty$.

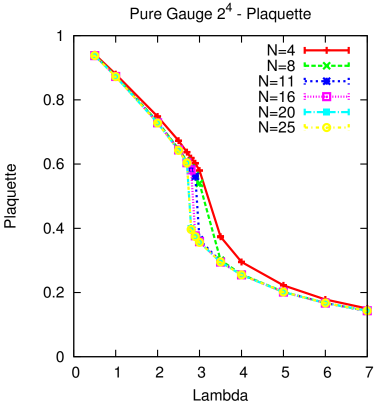
Conditions

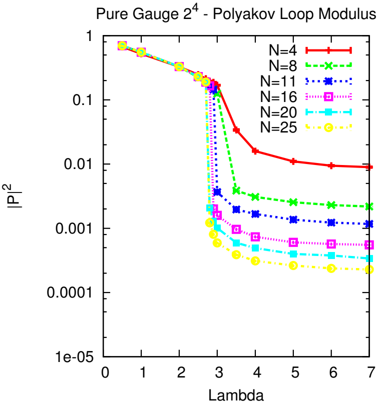
...but it turns out only

- for single–trace observables defined on the original lattice of side L, that are invariant under translations through multiples of the reduced lattice size L'
- and if the $U(1)^d$ center symmetry is not spontaneously broken, i.e. on the lattice the trace of the Polyakov loop vanishes.

Pure Gauge Phase Diagram

Pure Gauge: Plaquette and Polyakov loop vs λ.





Twisted Eguchi–Kawai

Gonzalez–Arroyo Okawa '83

Impose twisted boundary conditions, such that the classical minimum of the action preserves a Z_N^2 subgroup of the center symmetry.

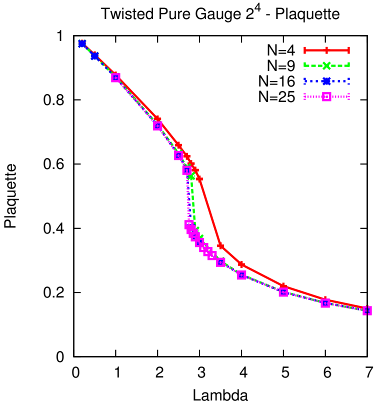
$$S_{TEK} = \frac{N_c}{\lambda} \sum_{\mu<\nu} Tr \left(z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger + h.c. \right)$$

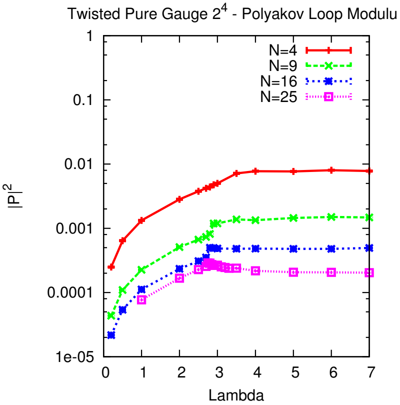
$$\text{where } z_{\mu\nu} = exp\{2\pi ik/\sqrt{N}\}$$

- Original choice is $k = 1$

Twisted Pure Gauge Phase Diagram

$k = 1$ Twisted Pure Gauge: Plaquette and Polyakov loop vs λ.





Twisted Eguchi–Kawai

- Original choice $k = 1$ seen to break center–symmetry at intermediate couplings for $NL^2 \gtrsim 100$
- But symmetry can be restored by scaling the twist k with N

Gonzalez–Arroyo Okawa [arXiv:1005.1981]

QCDadj

Kotvul Unsal Yaffe '07

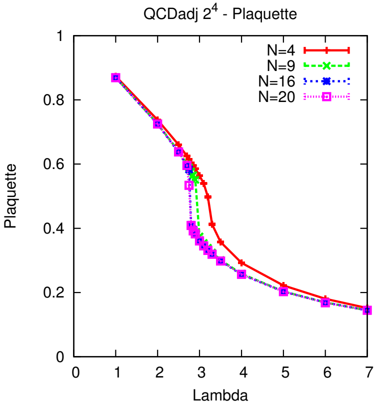
Add (massless or light) adjoint fermions with periodic boundary conditions

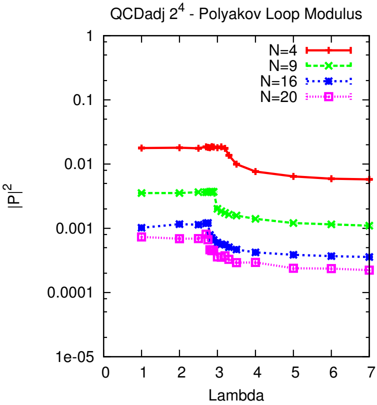
- Preserves center symmetry down to a single site
- and for a range of light adjoint fermion masses.

- Works in perturbation theory (for $am \lesssim \frac{1}{N}$)
- And in lattice simulations (possibly even for $am \lesssim 1$)

QCDadj Phase Diagram

QCDadj ($am_0 = 0$): Plaquette and Polyakov loop vs λ.





Why Large N?

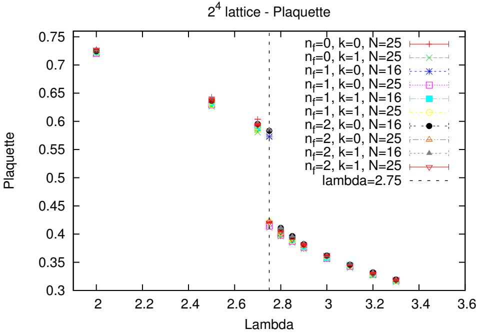
- In 2–loop perturbation theory, γ_* is independent of N, so we expect the large N value to be close to the $N = 2$ value.
- At large N the theory is (under certain conditions) volume independent, so the calculation can be done on a small lattice or even a single site.
- Interesting cross check of method, perturbation theory and large N volume independence.

Simulation Details

- Simulate QCDadj.
  - SU(N) gauge theory with 2 adjoint Dirac fermions with periodic boundary conditions.
- Use 2^4 lattices with N up to 25.
  - $V_{eff} \sim L^4 N^2$, so equivalent to $N \sim 100$ on a single site.
- Measure lowest 5% of eigenvalues of the Dirac operator Q^2.
  - Scales with N^2, ~ 400 eigenvalues for $N = 16$.
- Choose initial bare coupling $\lambda = 2.70$.
  - Need to stay in weak coupling phase.
  - But want fairly strong coupling to minimise $1/N$ effects.
- Simulate with and without the minimal symmetric twist.

Strong Coupling Phase

Want to choose a strong bare coupling, but need to stay on the physical side of the strong coupling transition; choose $\lambda = 2.70$



Method

At small eigenvalues, at leading order,

Spectral density of the Dirac Operator

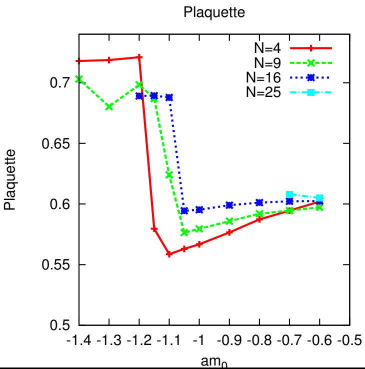
$$\rho(\omega) \propto \mu^{\frac{4\gamma_*}{1+\gamma_*}} \omega^{\frac{3-\gamma_*}{1+\gamma_*}} + \dots$$

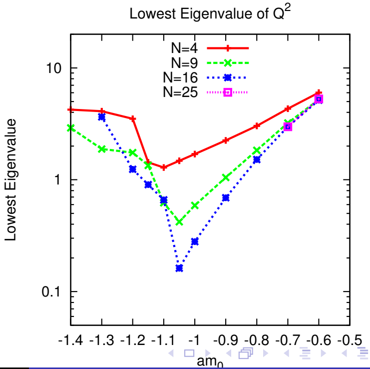
- Integral of this is the mode number, which is just counting the number of eigenvalues of the Dirac Operator on the lattice.
- Fitting this to the above form can give a precise value for γ, as done recently for MWT by Agostino Patella.

Patella [arXiv:1204.4432]

QCDadj: Bare Coupling and Critical Bare Mass

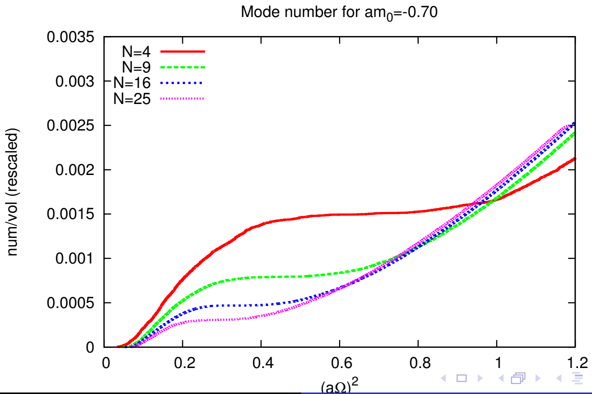
QCDadj at $\lambda = 2.70$: Scan of the plaquette and lowest Dirac operator eigenvalue as a function of the bare mass.
Want as light a mass as possible, while avoiding the Aoki phase.





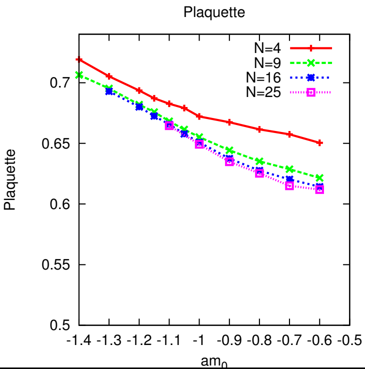
QCDadj Mode Number - Zero modes

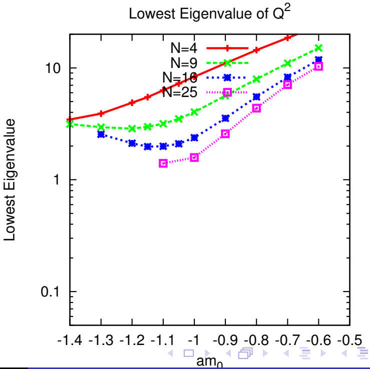
Finite volume effect: $4(N-1)$ would–be zero modes.
These are suppressed by the volume $(1/N^2)$ in the large–N limit.



QCDadj+Twist: Bare Coupling and Critical Bare Mass

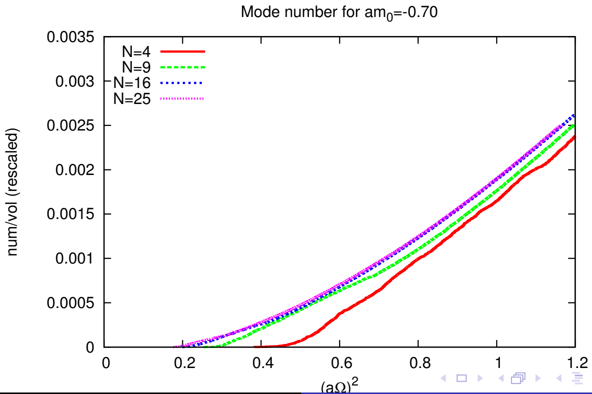
QCDadj+Twist at $\lambda = 2.70$; the lowest eigenvalue of the Dirac operator gives a similar critical bare mass, but do not see a discontinuity in the plaquette.





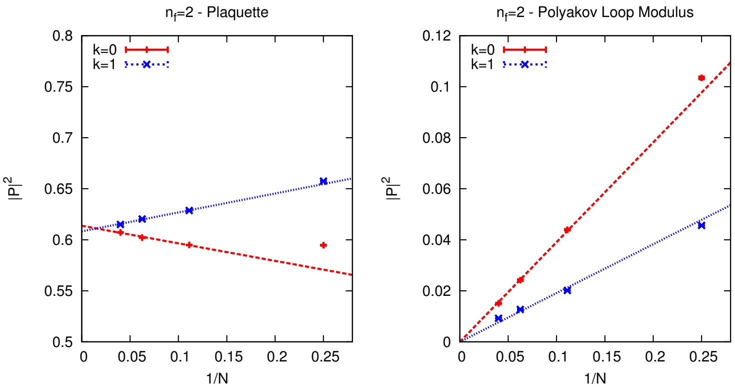
QCDadj+Twist Mode Number - N dependence

No would–be zero modes, they are suppressed by the twist.
Otherwise similar to the untwisted case.



Both twisted and untwisted

Comparison of twisted and untwisted case. $am_0 = -0.70$



Method

Fit data to the function

$$a^{-4}\overline{\nu}(\Omega) \simeq a^{-4}\overline{\nu}_0 + A\left[(a\Omega)^2 - (am)^2\right]^{\frac{2}{1+\gamma_*}}$$

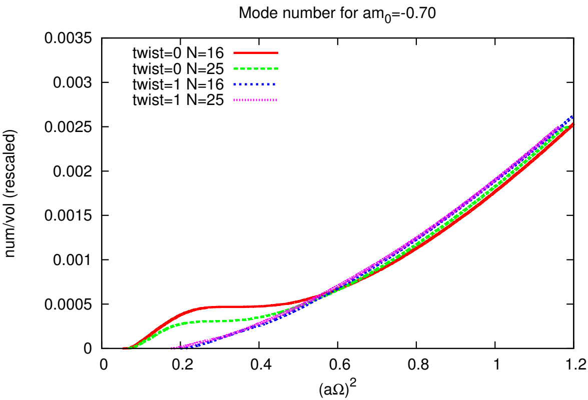
in some intermediate range $a\Omega_L < a\Omega < a\Omega_H$ where

- $a^{-4}\overline{\nu}(\Omega)$ is the number of eigenvalues of Q^2 below Ω^2 divided by the volume
- $a^{-4}\overline{\nu}_0$ is a fitted parameter (contribution of small excluded eigenvalues, $\propto M_{PS}^4$)
- am is a fitted parameter (physical mass)
- A is a fitted parameter

Patella [arXiv:1204.4432]

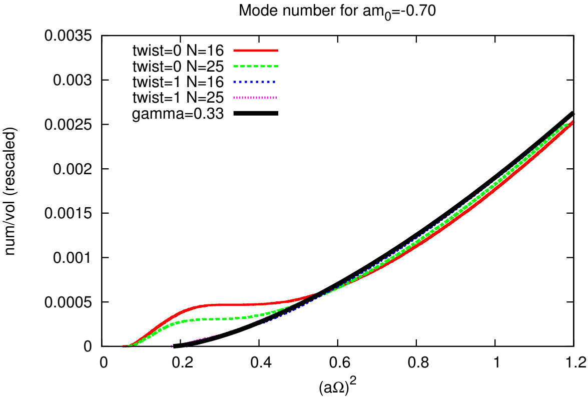
Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -0.70$



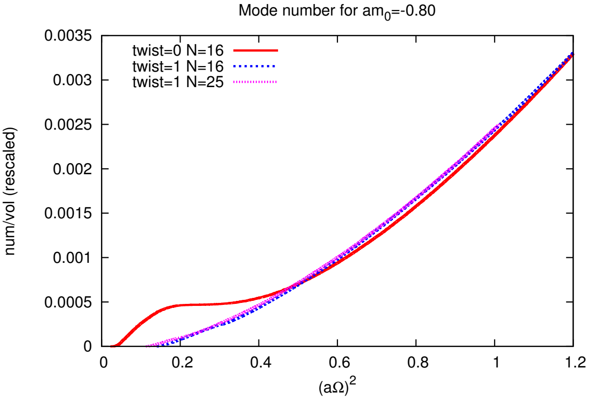
Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -0.70$



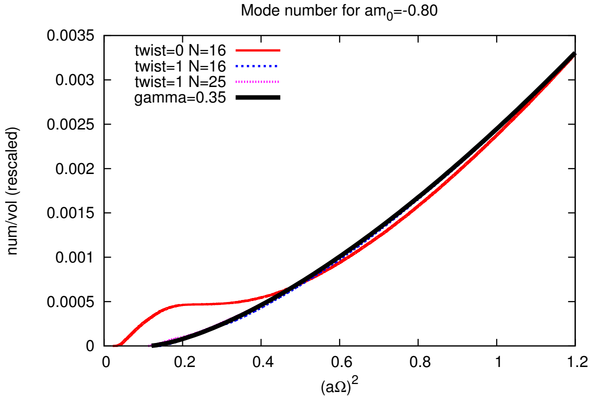
Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -0.80$



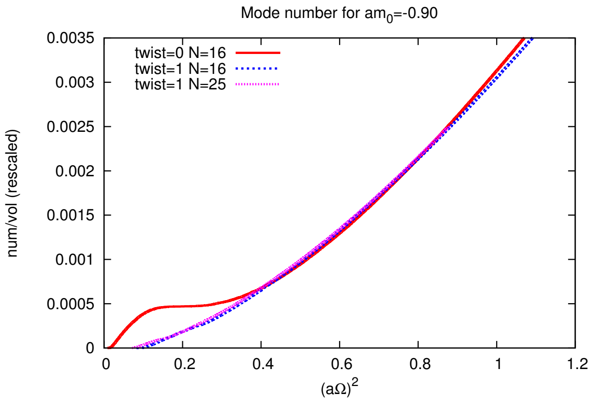
Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -0.80$



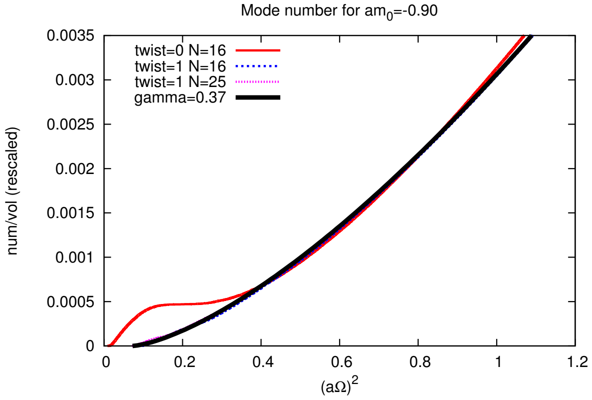
Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -0.90$



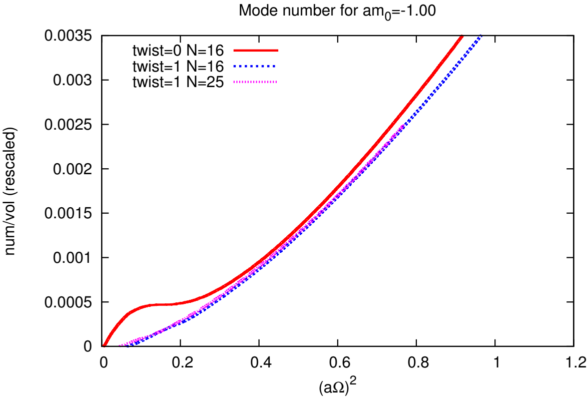
Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -0.90$



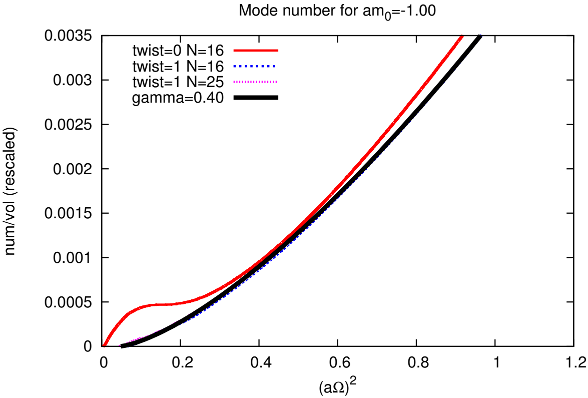
Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -1.00$

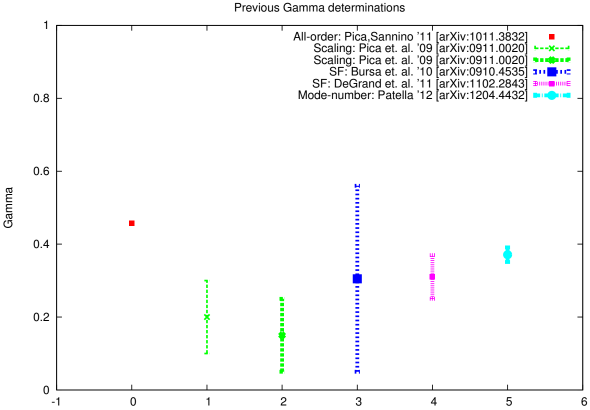


Mode Number - twist = 0,1

Comparison of twisted and untwisted case. $am_0 = -1.00$



Comparison with other results



Conclusion

- Promising initial results.
    - Volume reduction seems to work.
    - Eigenvalue spectrum looks sensible.
- Would be very interesting to compare with $n_f = 1$
- Also need to investigate the systematics of the fitting procedure.
- And want to try different twist and couplings, larger N, lighter masses.

Strong Coupling Phase

In the (unphysical) strong coupling phase, no zero modes, $1/N$ corrections are tiny, and quenched configurations, with or without twist, give the same Dirac operator spectrum as dynamical ones.