Walking Technicolor on the Lattice

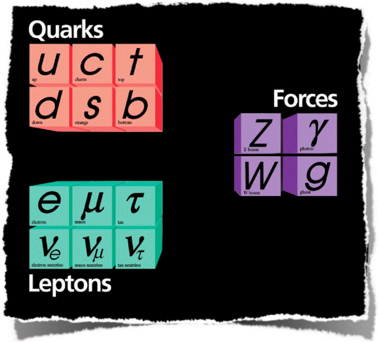
Liam Keegan

Nov 2010

Edinburgh University

SF: Francis Bursa, Luigi Del Debbio, Claudio Pica, Thomas Pickup
MCRG: Simon Catterall, Luigi Del Debbio, Joel Geidt

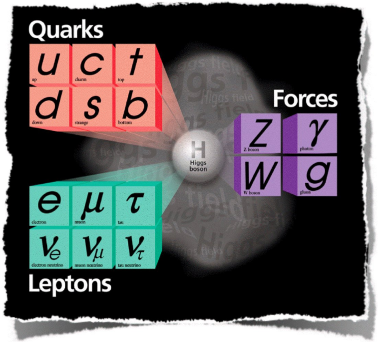
The Standard Model



Fermilab

- Standard Model is well verified experimentally
- Electroweak Symmetry breaking included (i.e. mass of W/Z bosons)
- But EWSB mechanism remains a mystery

The Higgs Mechanism



Fermilab

- Higgs mechanism will be tested at the LHC, but
  - Ad hoc: all fermion masses and mixings arbitrary parameters
  - Trivial: without new physics, Higgs decouples
  - Unnatural: quadratically sensitive to Planck scale, so requires fine tuning
- So thought to be an effective description of a more fundamental theory, e.g. SUSY, Technicolor, ...

Technicolor

- SM without Higgs already has some EW symmetry breaking.
- Quark condensate gives M_W of the order of the pion decay constant:

$$\langle \overline{u}_L u_R + \overline{d}_L d_R \rangle \neq 0 \rightarrow M_W = \frac{gF_\pi}{2} \sim 30 MeV$$

- So why not have some more 'techni-quarks' that form a condensate at a higher scale ($F_\pi^{TC} \sim 250 GeV \sim \Lambda_{TC}$)

Weinberg 78, Susskind 78

Extended Technicolor

- Add interactions between SM quarks and techni-quarks at some high scale Λ_{ETC}
- Get SM quark mass terms in effective low energy lagrangian:

Quark Masses

$$\frac{\langle\overline{\Psi}\Psi\rangle_{ETC}\overline{\psi}\psi}{\Lambda_{ETC}^2}$$

Dimopoulos, Susskind 79 - Eichten, Lane 80

Flavour Changing Neutral Currents

- But also get FCNC term:

Quark Masses

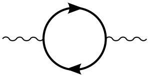
$$\frac{\langle\overline{\Psi}\Psi\rangle_{ETC}\overline{\psi}\psi}{\Lambda_{ETC}^2}$$

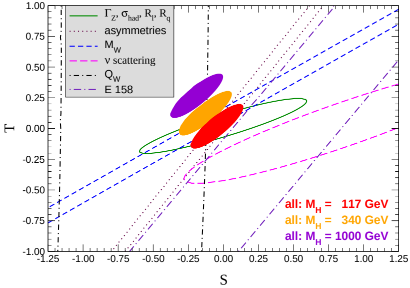
FCNC

$$\frac{\overline{\psi}\psi\overline{\psi}\psi}{\Lambda_{ETC}^2}$$

- Naively scaling up QCD leads to a problem:
- Need large $\Lambda_{ETC} \sim 1000\,TeV$ to suppress Flavour Changing Neutral Currents
- But this gives a strange quark mass that is ~ 50 times too small

S, T Parameters

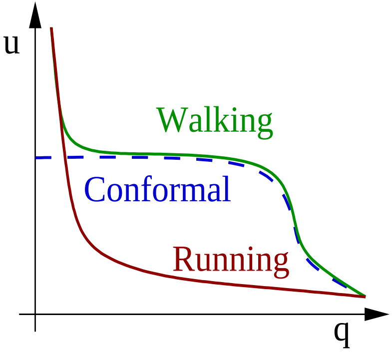


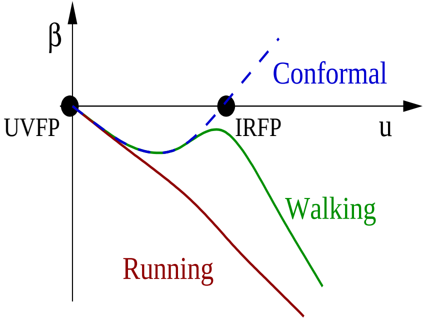


Particle Data Group 2008

- S,T parameters measure deviation from SM caused by new physics
- Naive QCD scaling gives $\sim 2\sigma$ disagreement with experiment
- Perturbative estimate: $S = \frac{1}{6\pi}\frac{N_f}{2}d(R) = 0.16$

Walking Technicolor Cartoon





Walking Technicolor Quark Masses

$$\langle \overline{\Psi}\Psi \rangle_{ETC} = \langle \overline{\Psi}\Psi \rangle_{TC} exp \left(\int_{\Lambda_{TC}}^{\Lambda_{ETC}} \gamma(\mu) d \ln \mu \right)$$

- In QCD this gives logarithmic enhancement:

$$\langle \overline{\Psi}\Psi \rangle_{ETC} = \log \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma} \langle \overline{\Psi}\Psi \rangle_{TC}$$

- But a walking coupling gives power enhancement:

$$\langle \overline{\Psi}\Psi \rangle_{ETC} = \left(\frac{\Lambda_{ETC}}{\Lambda_{TC}} \right)^{\gamma} \langle \overline{\Psi}\Psi \rangle_{TC}$$

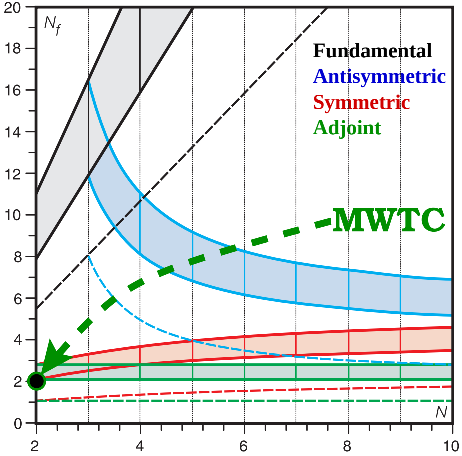
Walking Technicolor S Parameter

- Walking seems to reduce S parameter compared to running case.
- And other sectors of the theory, such as new leptons, are expected to contribute negatively

Dietrich, Sannino, Tuominen [arXiv:hep-ph/0505059]

- But ideally this also needs to be studied non-perturbatively

Phase Diagram



[arXiv:hep-ph/0611341]

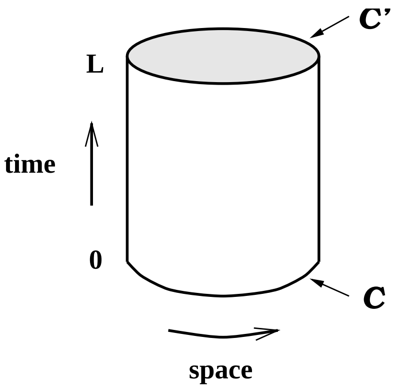
- MWTC: 2 dirac fermions transforming under the adjoint representation of SU(2)

Saninno, Tuominen
[arXiv:hep-ph/0405209]

Scheme dependence

- Walking/Running of coupling is scheme dependent
- Want to measure physical, scheme independent quantities:
  - **Existence** of fixed point
  - **Anomalous mass dimension** at the fixed point

Schrodinger Functional



(LxLxL box with periodic b.c.)

- Finite size renormalisation scheme
- Can be defined in continuum and on lattice
- Scale $\mu \sim 1/L$
- Dirichelet timelike bcs
- Constant gauge fields C, C'

Coupling

Boundary gauge fields induce a background chromoelectric field in the bulk with strength parametrised by η. ($\eta = \pi/4$)

Boundary gauge fields

$$\begin{aligned} U(x,k)|_{t=0} &= \exp\left[\eta\tau_3 a/iL\right] \\ U(x,k)|_{t=L} &= \exp\left[(\pi-\eta)\tau_3 a/iL\right] \end{aligned}$$

Define a coupling as the response of the system to perturbations of the background gauge field configuration.

SF Coupling

$$\overline{g}^2(L) = k\left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1}$$

Anomalous Dimension

The renormalised mass is given by

Renormalised Mass

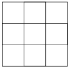
$$m = \frac{Z_A}{Z_P(L)} m_{PCAC}$$

We work at zero mass, and Z_A does not depend on L, so the running of the mass depends only on Z_P:

Pseudoscalar density renormalisation constant

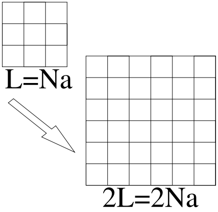
$$Z_P(L) = \frac{\sqrt{3f_1}}{f_P(L/2)}$$

Naive Scaling



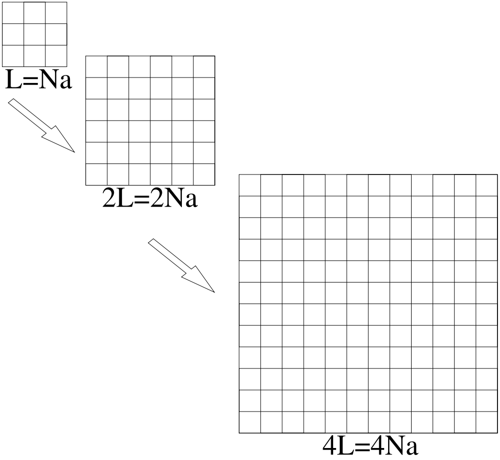
- Naive scaling: measure on $L, 2, 4L, \ldots, 2^n L$
- Corresponds to scales $\mu, \frac{1}{2}\mu, \frac{1}{4}\mu, \ldots, 2^{-n}\mu$
- But cpu time scales as $\sim N^5$, and we want to simulate over a large range ($\sim 10^3$) of scales
- So naive scaling method no good

Naive Scaling



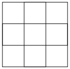
- Naive scaling: measure on $L, 2, 4L, \ldots, 2^n L$
- Corresponds to scales $\mu, \frac{1}{2}\mu, \frac{1}{4}\mu, \ldots, 2^{-n}\mu$
- But cpu time scales as $\sim N^5$, and we want to simulate over a large range ($\sim 10^3$) of scales
- So naive scaling method no good

Naive Scaling



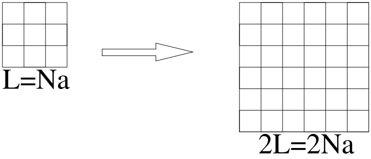
- Naive scaling: measure on $L, 2, 4L, \ldots, 2^n L$
- Corresponds to scales $\mu, \frac{1}{2}\mu, \frac{1}{4}\mu, \ldots, 2^{-n}\mu$
- But cpu time scales as $\sim N^5$, and we want to simulate over a large range ($\sim 10^3$) of scales
- So naive scaling method no good

Step Scaling



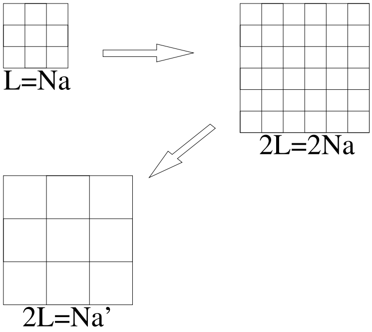
- Step scaling - only need $N^4, (2N)^4$
- $\overline{g}^2(\beta, L) = u$
- $u' = \overline{g}^2(\beta, 2L)$
- Now tune bare parameters until $\overline{g}^2(\beta', L) = u'$
- Repeat

Step Scaling



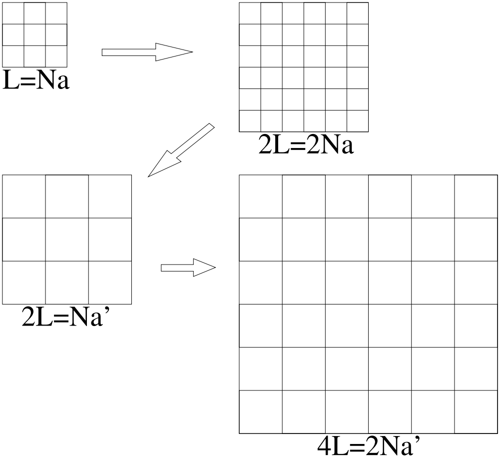
- Step scaling - only need $N^4, (2N)^4$
- $\overline{g}^2(\beta, L) = u$
- $u' = \overline{g}^2(\beta, 2L)$
- Now tune bare parameters until $\overline{g}^2(\beta', L) = u'$
- Repeat

Step Scaling



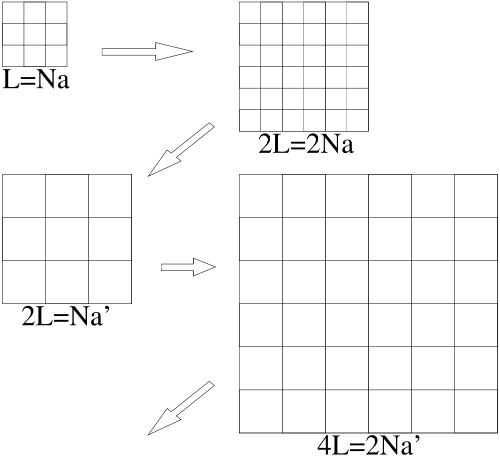
- Step scaling - only need $N^4, (2N)^4$
- $\overline{g}^2(\beta, L) = u$
- $u' = \overline{g}^2(\beta, 2L)$
- Now tune bare parameters until $\overline{g}^2(\beta', L) = u'$
- Repeat

Step Scaling



- Step scaling - only need $N^4, (2N)^4$
- $\overline{g}^2(\beta, L) = u$
- $u' = \overline{g}^2(\beta, 2L)$
- Now tune bare parameters until $\overline{g}^2(\beta', L) = u'$
- Repeat

Step Scaling



- Step scaling - only need $N^4, (2N)^4$
- $\overline{g}^2(\beta, L) = u$
- $u' = \overline{g}^2(\beta, 2L)$
- Now tune bare parameters until $\overline{g}^2(\beta', L) = u'$
- Repeat

Step Scaling

- This method was used by the ALPHA collaboration
- Can cover an arbitrary range of scales
- But each step requires retuning β, κ, which is time consuming
- And each step must be done sequentially, can't parallelise the runs

Bode et. al. [arXiv:hep-lat/0105003]

Interpolation Method

- Interpolation function method - just measure $\overline{g}^2$ at a range of β for each L and interpolate:

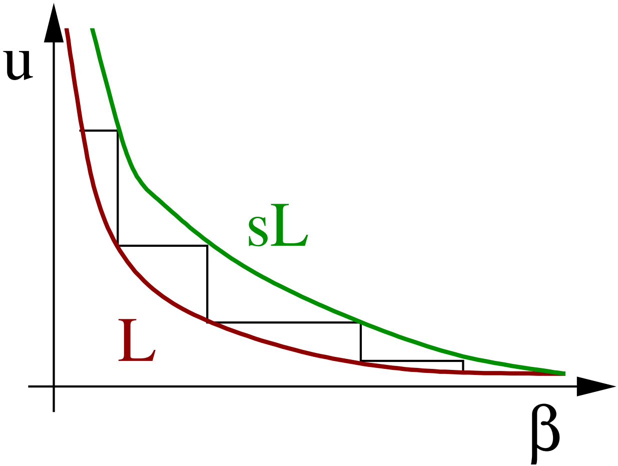
Coupling interpolation function

$$\frac{1}{\overline{g}^2(\beta, L/a)} = \frac{\beta}{2N} \sum_{i=0}^{n} c_i \left(\frac{2N}{\beta}\right)^i$$

- All simulations can be done in parallel, and no need for constant retuning
- However the choice of interpolation function introduces a new source of systematic error

method first used by Appelquist et. al. [arXiv:0901.3766]

Interpolation Method



Coupling Step Scaling Function

Lattice step scaling function

$$\Sigma(u, s, a/L) = \overline{g}^2(g_0, sL/a)\Big|_{\overline{g}^2(g_0, L/a)=u}$$

- Start on L^4 lattice where $\overline{g}^2 = u$
- Go to $(sL)^4$ lattice and measure $\overline{g}^2 = \Sigma$

Coupling Step Scaling Function

Continuum step scaling function

$$\sigma(u,s) = \lim_{a/L\to 0} \Sigma(u,s,a/L)$$

- Repeat for different lattice spacings a/L
- Extrapolate to the continuum $a/L \to 0$

Coupling Step Scaling Function

Relation to continuum beta-function

$$-2\log s = \int_u^{\sigma(u,s)} \frac{dx}{\sqrt{x}\beta(\sqrt{x})}$$

- Integrated β-function
- $\sigma(u,s) = u$ corresponds to a fixed point ($\beta = 0$)

Mass Step Scaling Function

Lattice step scaling function

$$\Sigma_P(u, s, a/L) = \left. \frac{Z_P(g_0, sL/a)}{Z_P(g_0, L/a)} \right|_{\bar{g}^2(L)=u}$$

- Start on L^4 lattice where $\bar{g}^2 = u$, measure Z_P
- Go to $(sL)^4$ lattice and measure new Z_P then take ratio

Mass Step Scaling Function

Continuum step scaling function

$$\sigma_P(u,s) = \lim_{a/L \to 0} \Sigma_P(u,s,a/L)$$

- Repeat for different lattice spacings a/L
- Extrapolate to the continuum $a/L \to 0$

Anomalous Dimension

Estimator for γ

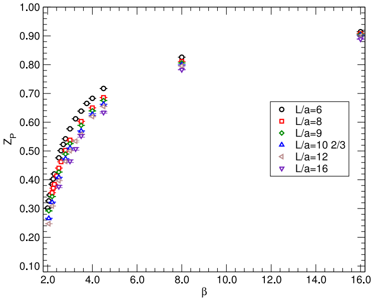
$$\hat{\gamma}(u) = -\frac{\log|\sigma_P(u,s)|}{\log|s|}$$

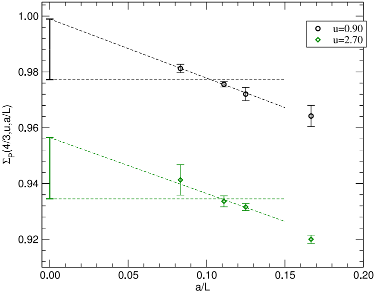
- At a fixed point this gives the anomalous dimension
- Away from a fixed point $\hat{\gamma}$ will deviate from γ

SF Simulation details

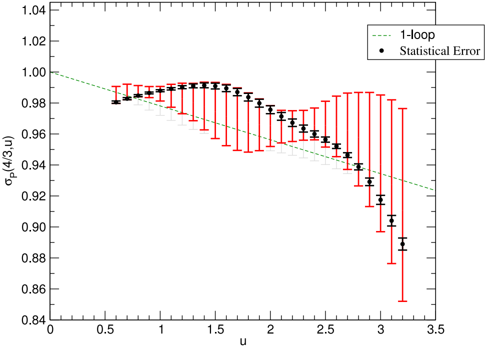
- Simulated on N^4 lattices where $N = 6, 8, 12, 16$
- β in range $2.0 - 16.0$
- Limited by bulk phase transition at $\beta \sim 2.0$
- Unimproved Wilson fermions
- Step size $s = 4/3$
- ~ 1000 configurations on the largest lattices

Z_P Data & Continuum Extrapolation



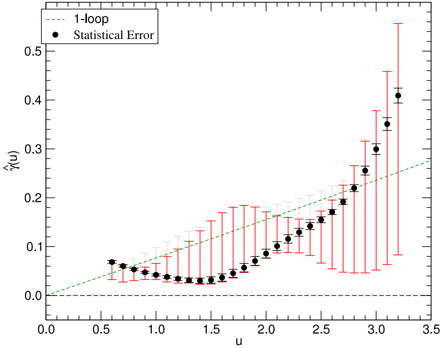


Sigma P



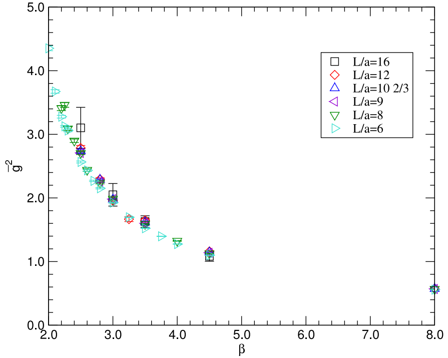
- Consistent with one-loop perturbative prediction

Mass Anomalous Dimension



- $\hat{\gamma}$ is well determined
- Consistent with one-loop prediction
- Smaller than desired for phenomenology
- But is sensitive to the location of the fixed point

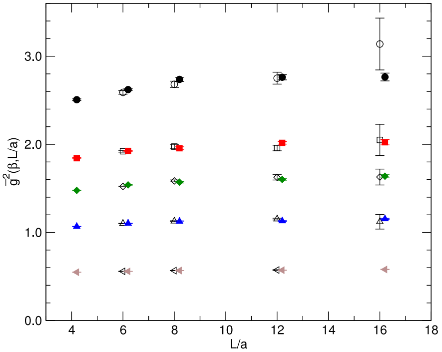
Coupling Data



- Not much variation with L
- Very good agreement with independent results

Hietanen, Rummukainen, Tuominen [arXiv:0904.0864]

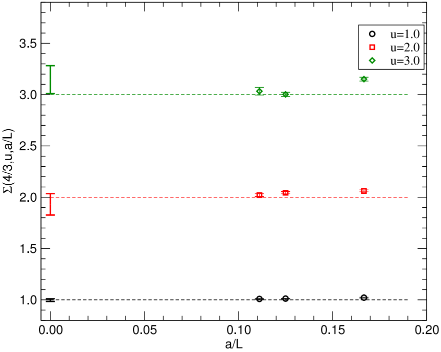
Coupling Data



- Not much variation with L
- Very good agreement with independent results

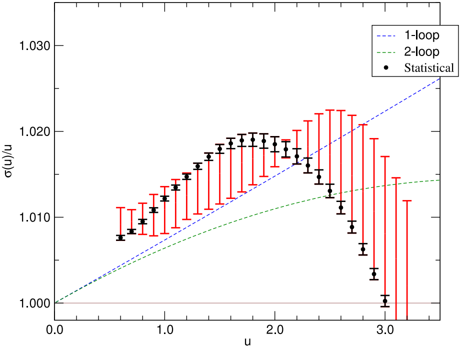
Hietanen, Rummukainen, Tuominen [arXiv:0904.0864]

Continuum Extrapolation



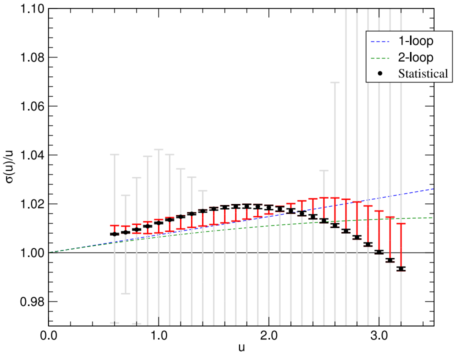
- No clear a/L dependence
- This is our largest source of error
- Continuum values consistent with no running within errors

Running Coupling



- Coupling runs very slowly
- Looks like there may be a fixed point at $u \sim 3$
- But once we include systematic errors the signal is swamped

Running Coupling



- Coupling runs very slowly
- Looks like there may be a fixed point at $u \sim 3$
- But once we include systematic errors the signal is swamped

SF Conclusion

- Have full control over statistical and systematic errors
- Can determine mass anomalous dimension well as a function of coupling
- But only scheme-independent at a fixed point
- In the region $2.0 < \overline{g}^2 < 3.2$ where there may be a fixed point we find $0.05 < \gamma < 0.56$

Wilson Renormalisation Group

- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

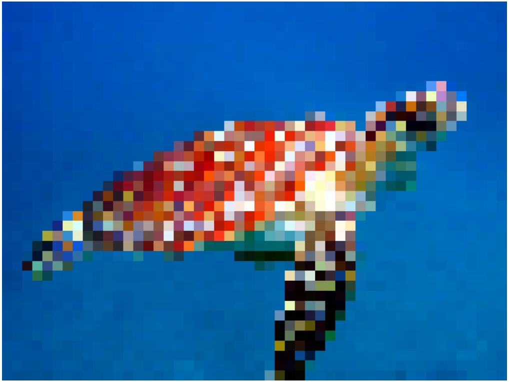
$$\hat{\xi}^{(0)} \quad , \quad \{g_i^{(0)}\}$$

Wilson Renormalisation Group

- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

$$\hat{\xi}^{(1)} = \hat{\xi}^{(0)}/2 \quad , \quad \{g_i^{(1)}\}$$

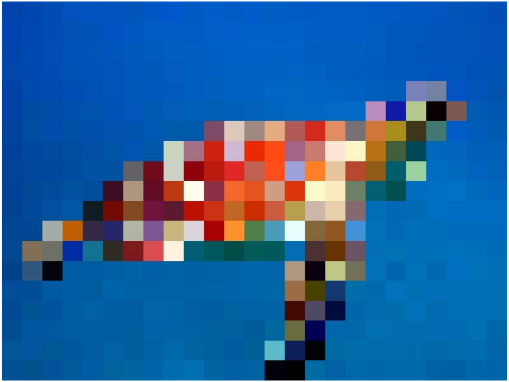
Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

$$\hat{\xi}^{(2)} = \hat{\xi}^{(0)}/2^2 \quad , \quad \{g_i^{(2)}\}$$

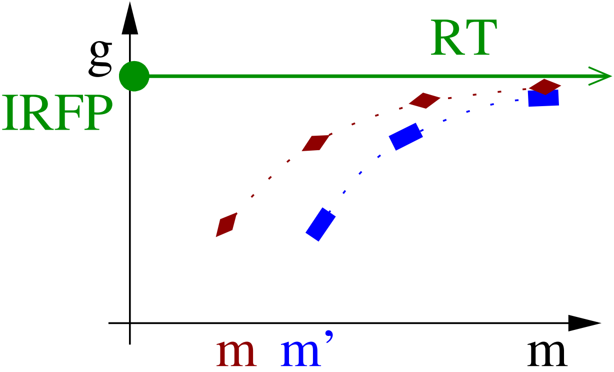
Wilson Renormalisation Group



- Spatially average locally / integrate out UV modes
- Leaves IR physics intact
- Look at evolution of all couplings

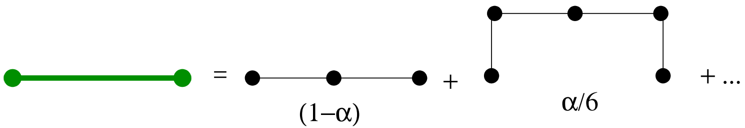
$$\hat{\xi}^{(3)} = \hat{\xi}^{(0)}/2^3 \quad , \quad \{g_i^{(3)}\}$$

Monte Carlo Renormalisation Group



- Match after $n(n-1)$ steps
- $s = 2$ change in scale
- Step scaling of bare couplings

Lattice Blocking Transform



- Free parameter α adjusts RG blocking transform
- Optimise α to approach RT quickly such that subsequent steps give the same matching

$$V_{n,\mu} = Proj\left[(1-\alpha)U_{n,\mu}U_{n+\mu,\mu} + \frac{\alpha}{6}\sum_{\nu\neq\mu} U_{n,\nu}U_{n+\nu,\mu}U_{n+\mu+\nu,\mu}U^{\dagger}_{n+2\mu,\nu}\right]$$

MCRG Key Points

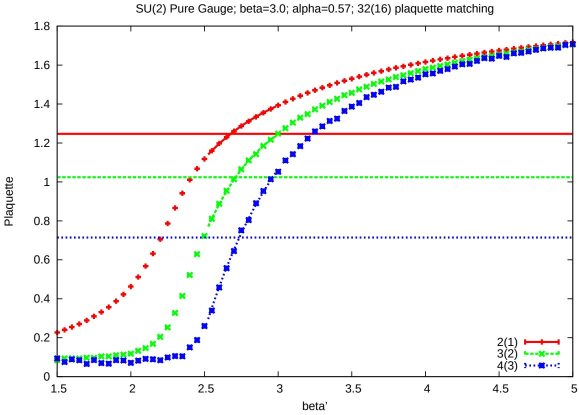
- Find pairs of couplings with identical blocked actions, whose correlation lengths differ by a factor 2
- Identify matching actions by comparing observables on blocked lattices (plaquette, 6-link and 8-link loops)
- Always match between lattices with the same number of points to minimise finite size errors
- Optimise α to approach the RT quickly so that subsequent steps give the same matching

Hasenfratz [arXiv:hep-lat/0907.0919]

Pure Gauge Simulation

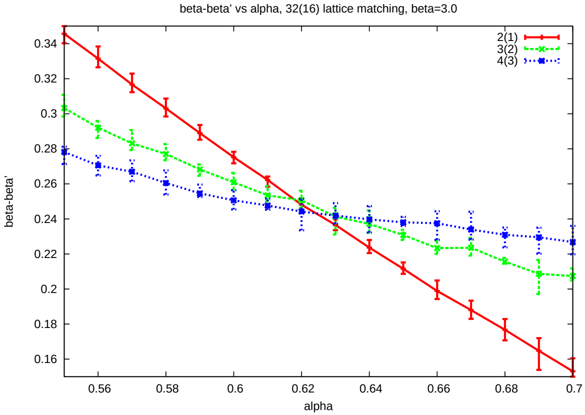
- Simulated on lattices of size L=32,16
- Allows for 3 matchings; 2(1), 3(2), 4(3) steps on the $32^4(16^4)$ lattices
- Optimise α such that these steps predict the same matching coupling
- Repeated this for three different blocking transforms, and on 16(8) lattices

Plaquette Matching



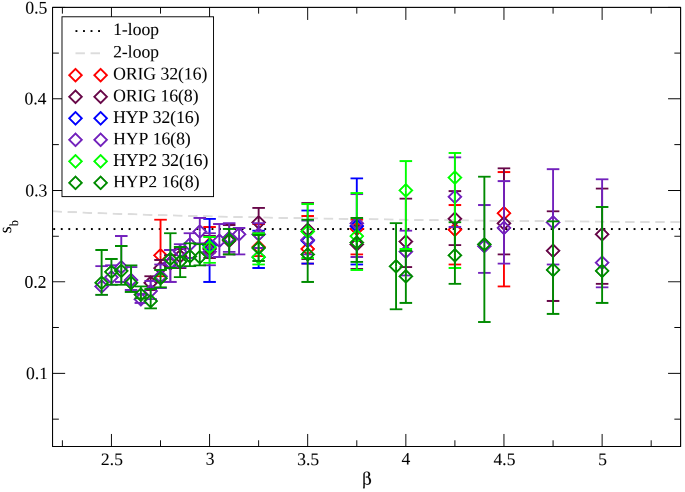
- 32(16) matching
- $\beta = 3.0$
- $\alpha = 0.57$

Alpha Optimisation

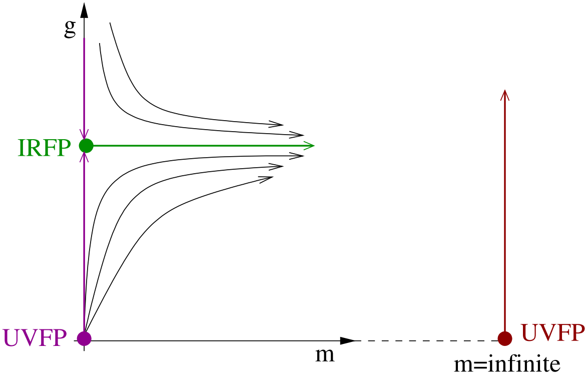


- $\beta = 3.0$
- $\alpha_{opt} \sim 0.63$
- $s_b = 0.24(2)$

Pure Gauge Bare Step Scaling

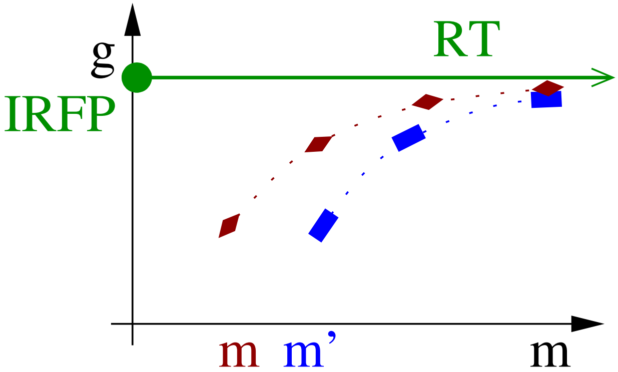


Phase diagram



- At IRFP:
- *m* relevant
- *g* irrelevant
- Near IRFP can match in m, value of g should be irrelevant

Phase diagram

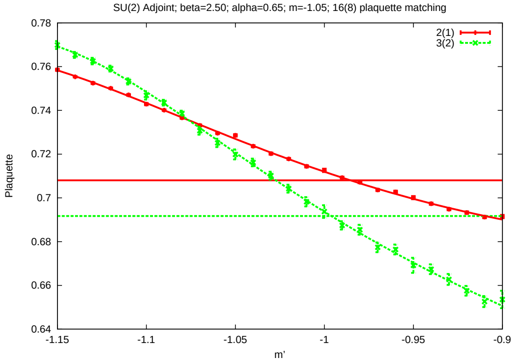


- At IRFP:
- *m* relevant
- *g* irrelevant
- Near IRFP can match in m, value of g should be irrelevant

Simulation details

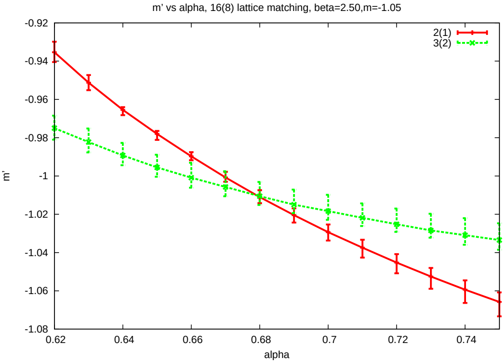
- Simulated on lattices of size L=16,8
- Allows for 2 matchings; 2(1), 3(2) steps on the $16^4(8^4)$ lattices
- Keep β constant, match in bare mass
- Optimise α such that these all agree to find continuum physics

Plaquette Matching



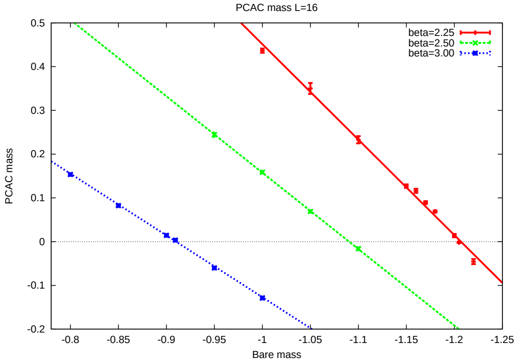
- 16^4 blocked two/three times
- Single mass $m = -1.05$
- 8^4 blocked one/two times
- Many masses $-1.15 < m' < -0.90$

Alpha Optimisation



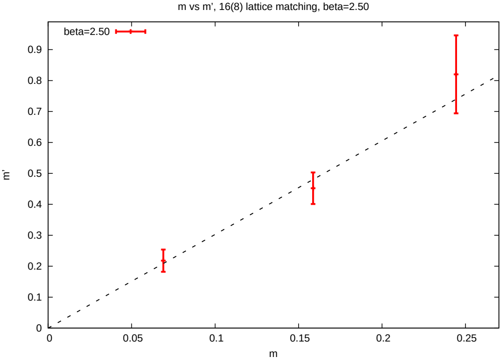
- $\alpha_{opt} \sim 0.68$
- $m = -1.05$
- $m' = -1.01(2)$

PCAC Masses



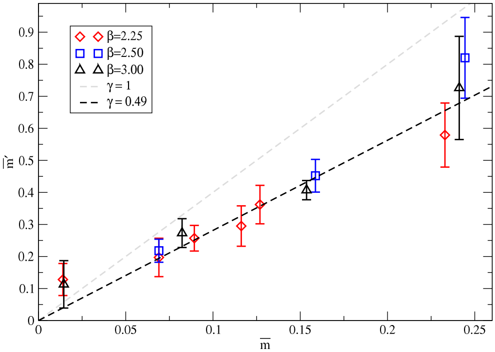
- Have matching bare masses, but additively renormalised quantities
- So need to convert to PCAC masses to be able to extract anomalous dimension

Anomalous Dimension



- Extract γ from ratio of masses:
- $m' = 2^{\gamma+1} m$
- To verify that beta is irrelevant, repeat at different beta...
- Linear fit gives $\gamma = 0.49(13)$

Anomalous Dimension



- Extract γ from ratio of masses:
- $m' = 2^{\gamma+1} m$
- To verify that beta is irrelevant, repeat at different beta...
- Linear fit gives $\gamma = 0.49(13)$

MCRG Conclusion

- Can determine mass anomalous dimension without having to measure the running of the coupling
- Independence of β strongly suggests we are at an IRFP
- We find $\gamma = 0.49(13)$

Summary

- Minimal Walking Technicolor requires a large $\gamma \sim 1$ anomalous dimension.
- We measure this quantity using two independent lattice techniques, and find consistent values:

SF

$0.05 < \gamma < 0.56$

[arXiv:0910.4535]

MCRG

$\gamma = 0.49(13)$

[arXiv:1010.5909]

- This is significantly smaller than desired for phenomenology.

Prediction for anomalous dimension

Conjectured all orders beta function

$$\beta(g) = \frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3} T(r) N_f \gamma(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G) \left(1 + \frac{2\beta_0'}{\beta_0}\right)}$$

$$\beta_0 = \frac{11}{3} C_2(G) - \frac{4}{3} T(r) N_f, \quad \beta_0' = C_2(G) - T(r) N_f$$

- For MWT this predicts anomalous dimension $\gamma = 3/4$ at fixed point, for CTC $\gamma = 5/3$
- This is a scheme-independent quantity at a fixed point

Ryttov, Sannino [arXiv:0711.3745]

Boundary Conditions

Boundary gauge fields

$$U(x,k)|_{t=0} = \exp\left[\eta\tau_3 a/iL\right]$$
$$U(x,k)|_{t=L} = \exp\left[(\pi-\eta)\tau_3 a/iL\right]$$

These induce a background chromoelectric field in the bulk with strength parametrised by η, we work at $\eta = \pi/4$.

Fermionic boundary conditions

$$P_+\psi = 0,\ \overline{\psi}P_- = 0 \quad \text{at} \quad t = 0$$
$$P_-\psi = 0,\ \overline{\psi}P_+ = 0 \quad \text{at} \quad t = L$$

These allow simulation directly at zero mass, $P_\pm = (1 \pm \gamma_0)/2$.

Coupling

Define a coupling as the response of the system to perturbations of the background gauge field configuration.

SF Coupling

$$\bar{g}^2(L) = k \left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1}$$

$$k = -24 \left(\frac{L}{a}\right)^2 \sin\left[\left(\frac{a}{L}\right)^2 (\pi - 2\eta)\right] \sim -12\pi$$

chosen such that $\bar{g}^2 = g_0^2$ to leading order in perturbation theory.

SF Coupling

- Choose background field B which is classical minimum of system, so fields close to B will dominate effective action

$$\Gamma[B] \equiv -\ln \mathcal{Z}[C, C'] = -\ln \left| \int D[\psi] D[\overline{\psi}] D[U] e^{-S} \right|$$

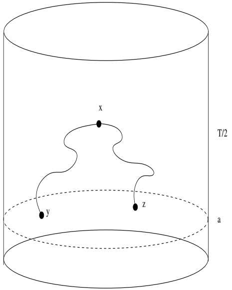
- Perturbative expansion

$$\Gamma[B] = \frac{1}{g_0^2} \Gamma_0[B] + \Gamma_1[B] + g_0^2 \Gamma_2[B] + \ldots$$

- Choose $\Gamma' \equiv \partial\Gamma / \partial\eta$ as observable, then can define a renormalised coupling as

$$\overline{g}^2 = \Gamma_0' / \Gamma' = k \left\langle \frac{\partial S}{\partial \eta} \right\rangle^{-1} = g_0^2 + \mathcal{O}(g_0^4)$$

PCAC Mass



SF bcs allow simulation directly at zero mass, which we define using the Partially Conserved Axial Current:

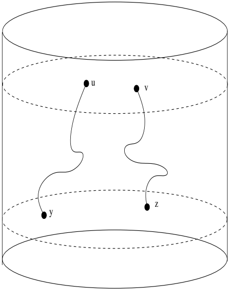
PCAC Mass

$$am(x_0) = \frac{\frac{1}{2}(\partial_0 + \partial_0^*) f_A(x_0)}{2 f_P(x_0)}$$

$$f_A(x_0) = -1/12 \int d^3y\, d^3z\, \langle \overline{\psi}(x_0)\gamma_0\gamma_5\tau^a\psi(x_0)\overline{\zeta}(y)\gamma_5\tau^a\zeta(z)\rangle$$

$$f_P(x_0) = -1/12 \int d^3y\, d^3z\, \langle \overline{\psi}(x_0)\gamma_5\tau^a\psi(x_0)\overline{\zeta}(y)\gamma_5\tau^a\zeta(z)\rangle$$

Z_P



Pseudoscalar density renormalisation constant

$$Z_P(L) = \frac{\sqrt{3f_1}}{f_P(L/2)}$$

$$f_1 = -1/12L^6 \int d^3u\, d^3v\, d^3y\, d^3z\, \langle \bar{\zeta}'(u)\gamma_5\tau^a\zeta'(v)\bar{\zeta}(y)\gamma_5\tau^a\zeta(z)\rangle$$

- f_1 correlator included to cancel boundary renormalisation factors

Mass Step Scaling Function

Relation to Beta-function

$$\sigma_P(u) = \left(\frac{u}{\sigma(u)}\right)^{(d_0/(2\beta_0))} \exp\left[\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dx \left(\frac{\gamma(x)}{\beta(x)} - \frac{d_0}{\beta_0 x}\right)\right]$$

Particle content of MWT

- Fermionic content:
  - (U,D) techni-quark doublet
  - (N,E) new lepton doublet
  - composite techniquark-technigluon doublet
- Composite Higgs from techni-pion

MWT LHC Phenomenology

- details depend on choice of ETC model
- then construct low energy EFT for LHC

Frandsen, Sannino, et. al. [arXiv:0710.4333v1] [arXiv:0809.0793v1]

MWT Dark Matter candidate

- lightest technibaryon is a cold dark matter candidate
- TIMP: Technicolour Interacting Massive Particle
- iTIMP: lightest weak isotriplet technibaryon
- Prospects for discovery/exclusion from both dark matter experiments and LHC

Frandsen, Sannino [arXiv:0911.1570]